APRIL-MAY 2025 EXAMINATION I B.TECH EXAMINATION MA 10011 MATHEMATICS-I

Time: 3 Hrs.]

[Max. Marks:70

TOTAL NO. OF QUESTIONS IN THIS PAPER: 5

Note: Attempt all the questions. All questions carry equal marks. Each question carries five subparts a, b, c, d and e. Attempt subparts a, b, c and any one from d or e in each question.

		MARKS	CO	BL	PI
(a)	Verify that $\omega_{xy} = \omega_{yx}$ when $\omega = xy^2 + x^2y^3 + x^3y^4$.	02	CO1	2	2.1.3
(b)	Find $\frac{dy}{dx}$ for the given implicit function	02	CO1	2	2.1.3
	$f(x,y) = x \sin(x-y) - (x+y) = 0$				
(c)	Verify $JJ^* = 1$ for the functions $x = u, y = u \ tanv, z = w$.	03	CO1	2	2.1.3
(d)	Find the degree of the function $u = tan^{-1} \left(\frac{y^2}{x} \right)$ and then	07	CO1	2	2.1.3
	prove that $(i) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} Sin 2u$				
	(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin^2 2u$				
	OR				
(e)	Obtain Taylor's expansion of $tan^{-1}\left(\frac{y}{x}\right)$ about (1, 1) upto and	07	CO1	3	1.1.2
	including second degree terms. Hence compute $f(1.1, 0.9)$.				
(a)	Define extreme values and saddle point of a function.	02	CO2	1	1.1.1
(b)	Find the radius of curvature at any point (s, ψ) of the curve $s = 8a \sin^2 \frac{\psi}{6}$.	02	CO2	2	2.1.3
(c)	Show that the equation of circle of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is	03	CO2	2	2.1.3
	$\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{a^2}{2}.$	0			
(d)	Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$ using Lagrange's method of undetermined multipliers.	07	CO2	2	2.1.3
	OR				
(e)	Find all the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$	07	CO2	2	2.1.3
(a)	Prove that $n\beta(m+1,n) = m\beta(m,n+1)$.	02	CO3	2	2.1.3
(b)	Evaluate $\Gamma(\frac{-5}{2})$.	02	CO3	2	2.1.3
	(c) (d) (e) (a) (e) (a)	 (c) Verify JJ* = 1 for the functions x = u, y = u tanv, z = w. (d) Find the degree of the function u = tan⁻¹ (y²/x) and then prove that (i) x ∂u/∂x + y ∂u/∂y = 1/2 Sin 2u (ii) x² ∂²u/∂x² + 2xy ∂²u/∂x∂y + y² ∂²u/∂y² = -Sin²u Sin2u OR (e) Obtain Taylor's expansion of tan⁻¹ (y/x) about (1, 1) upto and including second degree terms. Hence compute f(1.1, 0.9). (a) Define extreme values and saddle point of a function. (b) Find the radius of curvature at any point (s, ψ) of the curve s = 8a Sin² ψ/6. (c) Show that the equation of circle of curvature at the point ((x/4, a)/4) of the curve √x + √y = √a is (x - (3a/4))² + (y - (3a/4))² = a²/2. (d) Find the maximum value of x^myⁿz^p when x + y + z = a using Lagrange's method of undetermined multipliers. OR (e) Find all the asymptotes of the curve x³ + 2x²y - xy² - 2y³ + 4y² + 2xy + y - 1 = 0 (a) Prove that nβ(m + 1, n) = mβ(m, n + 1). 	(c) Verify $JJ^* = 1$ for the functions $x = u$, $y = u$ tanv, $z = w$. (d) Find the degree of the function $u = tan^{-1}\left(\frac{y^2}{x}\right)$ and then prove that (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2}Sin\ 2u$ (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -Sin^2u\ Sin2u$ OR (e) Obtain Taylor's expansion of $tan^{-1}\left(\frac{y}{x}\right)$ about $(1,1)$ upto and including second degree terms. Hence compute $f(1.1,0.9)$. (a) Define extreme values and saddle point of a function. 02 (b) Find the radius of curvature at any point (s, ψ) of the curve $s = 8a\ Sin^2\frac{\psi}{6}$. (c) Show that the equation of circle of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{a^2}{2}$. (d) Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$ using Lagrange's method of undetermined multipliers. OR (e) Find all the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$ (a) Prove that $n\beta(m+1,n) = m\beta(m,n+1)$.	(c) Verify $JJ^* = 1$ for the functions $x = u, y = u \ tanv, z = w$. (d) Find the degree of the function $u = tan^{-1} \left(\frac{y^2}{x}\right)$ and then prove that (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} Sin \ 2u$ (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -Sin^2 u \ Sin \ 2u$ (ii) Obtain Taylor's expansion of $tan^{-1} \left(\frac{y}{x}\right)$ about $(1,1)$ upto and including second degree terms. Hence compute $f(1.1,0.9)$. (a) Define extreme values and saddle point of a function. (b) Find the radius of curvature at any point (s, ψ) of the curve $s = 8a \ Sin^2 \frac{\psi}{6}$. (c) Show that the equation of circle of curvature at the point $\left(\frac{a}{a}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{a^2}{2}$. (d) Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$ using Lagrange's method of undetermined multipliers. OR (e) Find all the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$ (a) Prove that $n\beta(m+1,n) = m\beta(m,n+1)$.	(c) Verify $JJ^* = 1$ for the functions $x = u, y = u \ tanv, z = w$. (d) Find the degree of the function $u = tan^{-1}\left(\frac{y^2}{x}\right)$ and then prove that $(i) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} Sin \ 2u$ $(ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -Sin^2 u \ Sin \ 2u$ $(ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -Sin^2 u \ Sin \ 2u$ (e) Obtain Taylor's expansion of $tan^{-1}\left(\frac{y}{x}\right)$ about $(1,1)$ upto and including second degree terms. Hence compute $f(1.1,0.9)$. (a) Define extreme values and saddle point of a function. (b) Find the radius of curvature at any point (s,ψ) of the curve $s = 8a \ Sin^2 \frac{\psi}{6}$. (c) Show that the equation of circle of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{a^2}{2}$. (d) Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$ using Lagrange's method of undetermined multipliers. OR (e) Find all the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$ (a) Prove that $n\beta(m+1,n) = m\beta(m,n+1)$.

	(c)	Evaluate $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}} dx$.	03	CO3	2	2.1.3
	(d)	Evaluate $\iint r^3 dr d\theta$, over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.	07	CO3	2	2.1.3
		OR				
	(e)	Change the order of integration in the following integral and evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx$.	07	CO3	2	2.1.3
Q.4	(a)	Find the values of θ for which r attain its maximum and minimum values for the curve $r^2 = a^2 \cos 2\theta$.	02	CO4	2	1.1.1
	(b)	Write down the formula to calculate area and length of the curve $y = f(x)$ between $x = a$ and $x = b$.	02	CO4	1	1.1.1
	(c)	Find the length of the arc f the parabola $y^2 = 8x$ cut off by its latus rectum.	03	CO4	2	1.1.1
	(d)	Trace the cardioid $r = a(1 - \cos \theta)$.	07	CO4	2	2.1.3
	1	OR				
	(e)	Show that the volume of the spindle solid generated by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the X-axis is $\frac{32\pi a^3}{105}$.	07	CO4	2	2.1.3
Q.5	(a)	The mean of 200 items was 50. Later on it was discovered that two items were misread as 92 and 8 instead of 192 and 88 respectively. Find out the correct mean.	02	CO5	3	1.1.1
	(b)	Write a short note on correlation.	02	CO5	1	2.4.1
	(c)	Find the Mode from the following data: Age 0-6 6-12 12-18 18-24 24-30 30-36 36-42 Frequency 6 11 25 35 18 12 6	03	CO5	2	2.4.1
	(d)	Fit a second degree Parabola to the following data: X: 10 15 20 25 30 35 40 Y: 11 13 16 20 27 34 41	07	CO5	2	2.4.1
		OR		005	2	2 4 1
	(e)	following data:	07	CO5	3	2.4.1
		Roll 1 2 3 4 5 6 7 8 9 10 A: 78 36 98 25 75 82 90 62 65 39 B: 84 51 91 60 68 62 86 58 33 47				×
