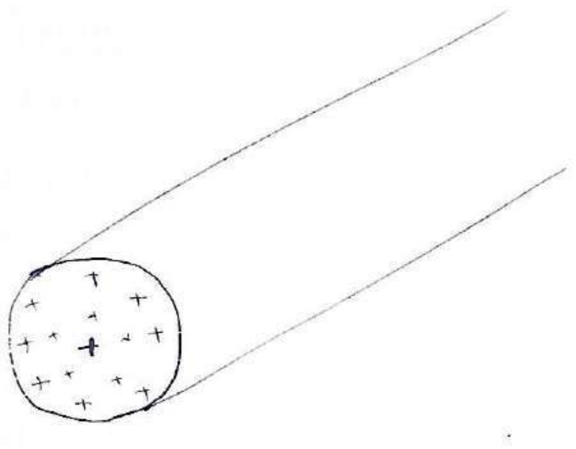
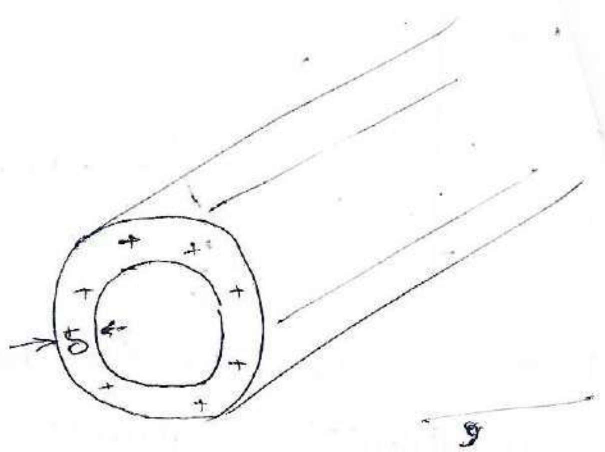
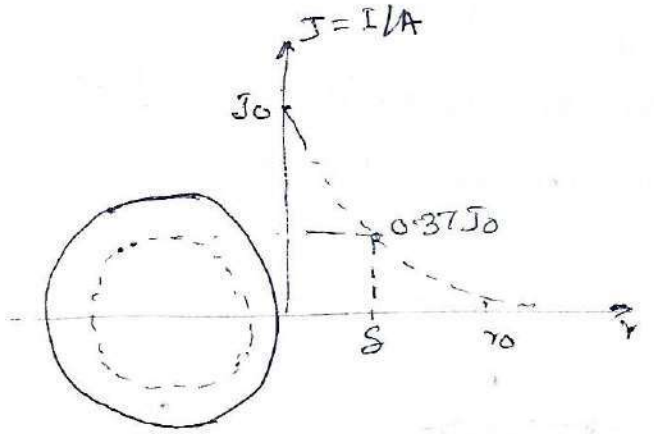
- Skin Effect -



DC



CH.



$\uparrow R = \frac{\rho L}{A} \downarrow$

At

Consider Cu as Cond. medium, what is skin depth at 60 Hz & 1 MHz

$\mu = 4\pi \times 10^{-7}$ H/m.

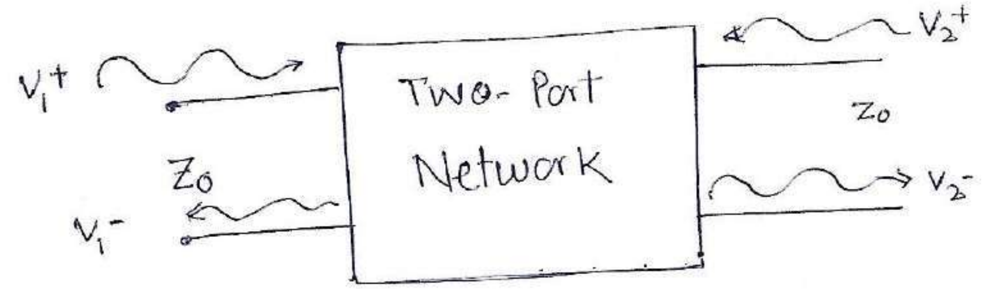
$\sigma = 5.8 \times 10^{7}$ S/m.

at f = 60, $\delta = 0.85$ cm.

1 MHz $\delta = 0.007$ cm.

Skin depth (δ): Current density falls off exponentially from surface of conductor towards center.
At Critical depth (δ) called as skin depth or depth of penetration, signal amplitude is $1/e$ or 36.8% of its surface amplitude. $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

Scattering Parameters



To define the S-parameters accurately, we will Consider a voltage phasor $[V_i^+]$ incident on & a voltage phasor $[V_i^-]$ reflected from the terminals of two-port network $(i=1,2)$ as shown in Fig.

The Scattering matrix is now defined to describe the linear relationship betn. incident voltage wave phasor matrix $[V_i^+]$ & reflected wave phasor matrix $[V_i^-]$.

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

$$\text{or} \quad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

where $[V_i^-] = [S][V^+]$ $\quad [V^-] = \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix}$, $[V^+] = \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$S_{11} = \left.\dfrac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma_{IN}$ Input reflection coefficient when o/p port is terminated in a matched load

$S_{21} = \left.\dfrac{V_2^-}{V_1^+}\right|_{V_2^+=0}$ Forward transmission Coefficient when o/p port is terminated in a matched load

$S_{12} = \left.\dfrac{V_1^-}{V_2^+}\right|_{V_1^+=0}$ Reverse transmission Coefficient when i/p port is terminated in a matched lo[ad]

$S_{22} = \dfrac{V_2^-}{\ldots}\Big| = \Gamma_{out}$ o/p reflection Coefficient when i/p port is terminated in a matched load.

Properties of S-parameters.

1) Reciprocal Networks. – A reciprocal N/w is defined as N/w that satisfies the reciprocity theorem.

For all reciprocal N/w's [S] matrix is symmetrical.

i.e. $S_{12} = S_{21}$

2) A symmetrical reciprocal N/ws. – For all symmetrical reciprocal N/ws [S] matrix is –

$$S_{11} = S_{22}$$
$$S_{12} = S_{21}.$$

3) Loss-less Networks

Unity property of S-matrix. – This property states that for a passive N/w lossless N-port N/w, the sum of the products of each term of any one row multiplied by its own complex conjugate is unity –

$$S_{11} \cdot S_{11}^* + S_{21} \cdot S_{21}^* = 1$$
$$S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* = 1$$

if lossless N/w is also reciprocal is $S_{12} = S_{21}$

$S_{12} = S_{21}$, $|S_{11}| = |S_{22}|$, $|S_{11}|^2 + |S_{21}|^2 = 1$

Zero property of S-matrix – This property states that for a passive N/w the sum of products of each term of any row multiplied by complex conjugate of the corresponding terms of any other row is zero.

$$S_{11} \cdot S_{12}^* + S_{21} \cdot S_{22}^* = 0$$
$$S_{12} \cdot S_{11}^* + S_{22} \cdot S_{21}^* = 0.$$

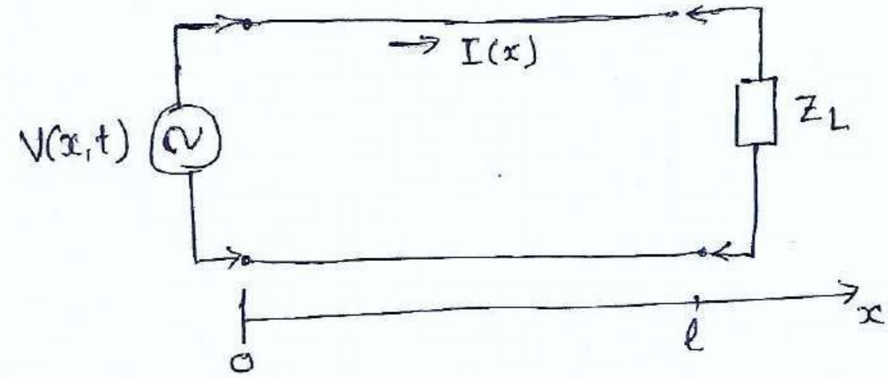
is lossless N/w is also reciprocal (ie. $S_{12} = S_{21}$)

$$S_{12} = S_{21}$$
$$S_{11} \cdot S_{21}^* = S_{21} \cdot S_{22}^* = 0 \qquad |S_{11}| = |S_{21}|$$

EI 37503/EI37513: High Frequency Engineering

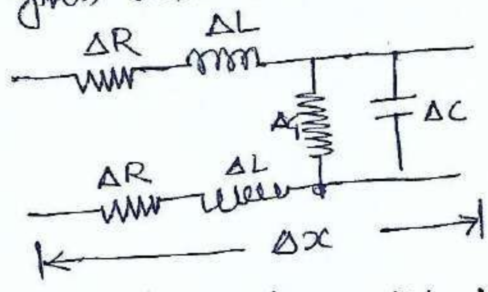
Unit-II - Transmission Line Theory.

Let us Consider two wires running parallel to each oth
having length 'l' as shown below.

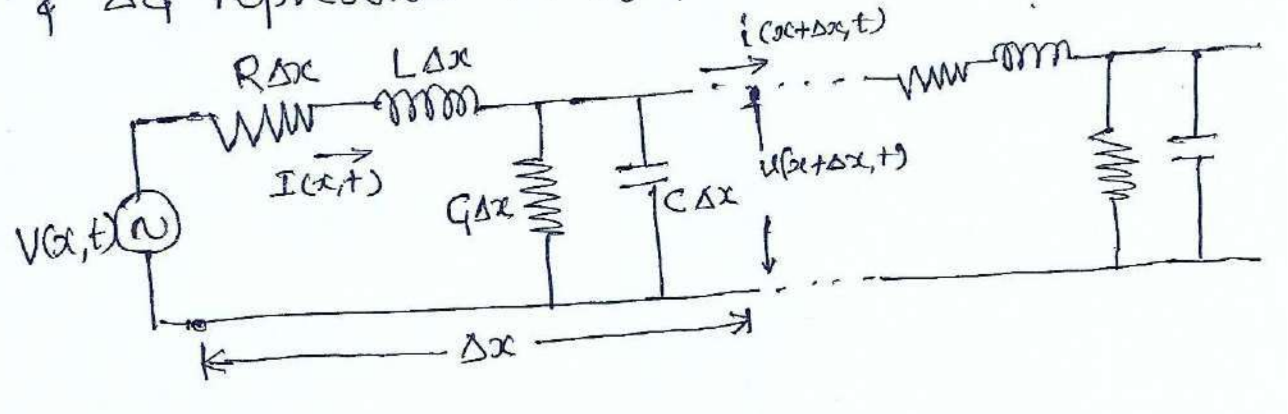


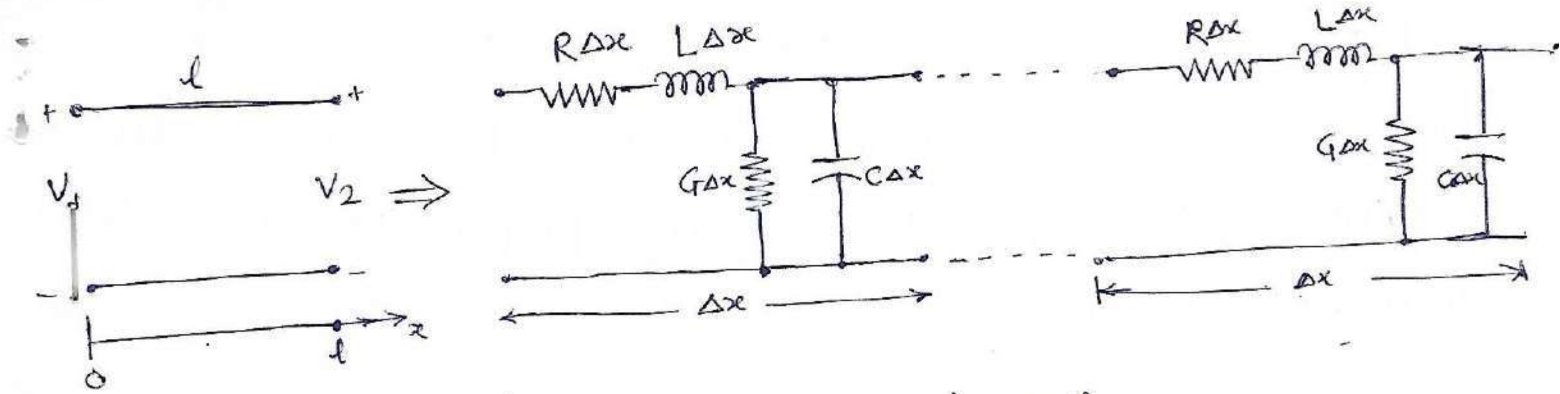
In order to find current $I(x)$ & Voltage $V(x)$ at any point x, it
required to divide or make sub parts of Δx such that the
Wavelength 'λ' should be Comparable to length Δx of T.L. so that
KVL & KCL can be applied to find $V(x)$ & $I(x)$

Now the equivalent circuit of Δx can be draw
as given below:

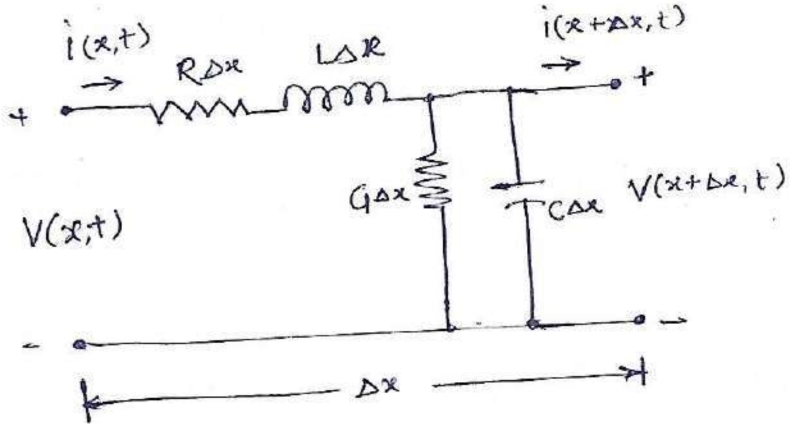


ΔR represents skin resistance of wire, ΔL represents the
self inductance of wire, ΔC represents capacitance between tw
wires & ΔG represents leakage, which is represented by Condu





As a thumb rule, when Ave. size of the discrete ckt component is more than a tenth of wavelength, the transmission line theory should be applied: $l_A \geq \lambda/10$.



At high frequencies, a transmission line can be modeled as shown above.

To develop the governing differential equations, we will examine one Δx section of a transmission line as shown in Fig (b).

Using KVL for Δx section, we can write

$$v(x,t) = i(x,t)\cdot R\Delta x + L\Delta x \cdot \partial i(x,t)/\partial t + v(x+\Delta x,t) \quad -(1)$$

Upon rearranging & dividing both sides by Δx, we obtain

$$-\frac{v(x+\Delta x,t) - v(x,t)}{\Delta x} = R\cdot i(x,t) + L\frac{\partial i(x,t)}{\partial t} \quad -(2)$$

Letting $\Delta x \to 0$, gives

$$-\frac{\partial v(x,t)}{\partial x} = R\cdot i(x,t) + L\frac{\partial i(x,t)}{\partial t} \quad -(3)$$

Similarly, using KCL we can write

$$i(x,t) = v(x+\Delta x, t)\cdot G\Delta x + C\Delta x\frac{\partial v(x+\Delta x,t)}{\partial t} + i(x+\Delta x,t) \quad -(4)$$

Upon rearranging terms, dividing by Δx & letting $\Delta x \to 0$ gives.

$$-\frac{\partial i(x,t)}{\partial x} = G\cdot V(x,t) + C\frac{\partial v(x,t)}{\partial t} \quad -(5)$$

Eqns (3) & (5) are cross-coupled equations in terms of i &
These eqns can be separated by first differentiating bo
equations with respect to x & then properly substituting for
terms, which leads to —

$$-\frac{\partial^2 v(x,t)}{\partial x^2} = R \cdot \frac{\partial i(x,t)}{\partial x} + L \frac{\partial^2 i(x,t)}{\partial x \cdot \partial t}$$

$$= -R\left[G \cdot v(x,t) + C \cdot \frac{\partial v(x,t)}{\partial t}\right] - L\left[G \frac{\partial v(x,t)}{\partial t} + C \cdot \frac{\partial^2 v(x,t)}{\partial t^2}\right]$$

or

$$\frac{\partial^2 v(x,t)}{\partial x^2} = LC \frac{\partial^2 v(x,t)}{\partial t^2} + (RC + LG) \cdot \frac{\partial v(x,t)}{\partial t} + RG \cdot v(x,t) \quad — (6)$$

Similarly for i we can get.

$$\frac{\partial^2 i(x,t)}{\partial x^2} = LC \cdot \frac{\partial^2 i(x,t)}{\partial t^2} + (RC + LG) \cdot \frac{\partial i(x,t)}{\partial t} + RG\, i(x,t) \quad \leftarrow (7)$$

For sinusoidal signal variation for V & I, we can write correspond
phasors as follows.

$$v(x,t) = Re\left[V(x) \cdot e^{j\omega t}\right]$$

$$I(x,t) = Re\left[I(x) \cdot e^{j\omega t}\right]$$

Differentiating a time domain sinusoid $dx(t)/dt$ amounts to the
multiplication of the corresponding phasor (A) by $j\omega$.
Using this property eq. (6) & (7) can be written as —

$$\frac{d^2 V(x)}{dx^2} - \gamma^2 V(x) = 0$$

$$\frac{d^2 I(x)}{dx^2} - \gamma^2 I(x) = 0$$

Where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$