Consider a 50Ω lossless transmission line of length $l = 1m$, connected to a generator operating at $f = 1GHz$ & having $V_g = 10V$ with $Z_g = 50\Omega$ at one end & connected to a load $Z_L = 100\Omega$ at the other end.

Determine

a) The voltage & current at any point on the transmission line.

b) The voltage at the generator (V_i) & load (V_L) ends.

c) The reflection coefficient at any point on the line.

Sol^n

$$Z_g = Z_0 = 50\Omega$$

$$\Gamma_g = 0 \quad \text{but } Z_L \neq Z_0$$

$$\therefore \Gamma_L \neq 0$$

$$\beta = \frac{\omega}{c} = \frac{2\pi \times 1 \times 10^9}{3 \times 10^8}$$

$$\beta = \frac{2\pi}{\lambda} \text{ but } \lambda = \frac{c}{f}$$

$$\therefore \beta = \frac{2\pi f}{c} = \frac{\omega}{c}$$

$$= \frac{20\pi}{3} \text{ rad/m.}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-50}{100+50} = \frac{1}{3}$$

$$V(x) = \frac{V_g}{2} e^{-j\beta(x+l)} \cdot (1 + \Gamma_L e^{j2\beta x}) = 5 e^{-j20\pi(x+1)/3} \left[1 + \frac{1}{3} e^{j40\pi x/3}\right]$$

$$I(x) = \frac{V_g}{2Z_0} e^{-j\beta(x+l)} \cdot (1 - \Gamma_L e^{j2\beta x}) = \frac{10}{2 \times 50} e^{-j20\pi(x+1)/3} \left[1 - \frac{1}{3} e^{j40\pi x/3}\right]$$

$$= 0.1 e^{-j20\pi(x+1)/3} \left(1 - \frac{1}{3} e^{j40\pi x/3}\right)$$

b) At gen. end $x = -1m$

$$V_i = V(-1) = 5\left(1 + \frac{1}{3} e^{-j40\pi/3}\right) = -4.16 + j1.44$$

At load end $x = 0$

$$V_L = V(0) = 5 e^{-j20\pi/3} \left(1 + \frac{1}{3}\right) = \frac{20}{3} e^{-j20\pi/3}$$

c) Ref. coeff. at any pt. $\Gamma(x) = \Gamma_L e^{j2\beta x} = \frac{1}{3} e^{j40\pi x/3}$

$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 1/3}{1 - 1/3} = 2$$

Derivation of expressions for α & β in terms of primary constant (R, L, G, C)

→ The propagation constant (γ), attenuation constant (α), phase constant (β) & characteristic impedance (Z_0) are called secondary coefficient or secondary constants of transmission line.

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta \quad -(1)$$

by squaring both sides.

$$\alpha^2 + (j\beta)^2 + 2j\alpha\beta = (R+j\omega L)(G+j\omega C)$$
$$= RG + j\omega CR + j\omega LG - \omega^2 LC$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = RG - \omega^2 LC + j\omega(LG+RC) \quad -(2)$$

Equating real & imaginary parts.

$$\alpha^2 - \beta^2 = (RG - \omega^2 LC) \quad -(3)$$
$$2\alpha\beta = \omega(LG+RC) \quad -(4)$$

Also we can write.

$$|\gamma| = \sqrt{\alpha^2+\beta^2} = \sqrt{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} \quad -(5)$$

Squaring both sides.

$$\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \quad -(6)$$

Adding eq. (3) & (6)

$$2\alpha^2 = (RG-\omega^2LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\boxed{\alpha = \pm\sqrt{\frac{1}{2}\left[(RG-\omega^2LC)\right] + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}}$$

Subtracting eq. (8) from eq (3)

$$-2\beta^2 = (RG - \omega^2 LC) - \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\beta^2 = -\frac{1}{2}\left[(RG - \omega^2 LC) - \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]$$

$$\beta^2 = \frac{1}{2}\left[-(RG - \omega^2 LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]$$

$$\boxed{\beta = \pm\sqrt{\frac{1}{2}\left[(\omega^2LC - RG) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}}$$

$$V_p = \frac{\omega}{\beta}$$

$$\beta^2 = \omega^2 K^2 = \frac{1}{2}(\omega^2LC - RG) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2}.$$

Distortions in Transmission Lines

Distortion is said to occur when the frequencies of s are attenuated by different amounts or different frequency components of a complex voltage wave experience different amounts of phase shifts.

Distortions in transmission line are of two types:

(1) Frequency Distortion (2) Delay Distortion.

1. Frequency Distortion - When various frequency component of signal are attenuated by different amounts then frequency distortion is said to occur. When the attenuation constant 'α' is not a function of frequency, there is no frequency distortion.

$$\alpha = \sqrt{\frac{1}{2}(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}$$

To eliminate frequency distortion

$$\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} = \omega^2 LC + K$$

Squaring both sides:

$$(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2) = \omega^4 L^2 C^2 + K^2 + 2\omega^2 LCK$$

$$R^2 G^2 + \omega^4 L^2 C^2 + \omega^2 (L^2 G^2 + R^2 C^2) = \omega^4 L^2 C^2 + 2\omega^2 LCK + K^2$$

Comparing coefficients of ω^2 & the constant terms

$$\boxed{K = RG} \quad \& \quad L^2 G^2 + R^2 C^2 = 2KLC$$

$$L^2 G^2 + R^2 C^2 = 2RGLC$$

$$\therefore (LG - RC)^2 = 0 \quad \text{or} \quad LG = RC$$

$$\boxed{R/L = G/C}$$

2. Delay Distortion :- When various frequency components arrive at different times (delay is not constant) then delay distortion or phase distortion is said to occur. When the phase velocity is independent of frequency or phase constant β is multiplied by ω, there is no delay distortion or phase distortion.

$$\beta = \sqrt{\frac{1}{2}(\omega^2 LC - RG) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}$$

To eliminate delay distortion

$$\beta = \omega K = \sqrt{\frac{1}{2}(\omega^2 LC - RG) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}$$

Squaring both sides.

$$\omega^2 K^2 = \frac{1}{2}(\omega^2 LC - RG) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}$$

$$2\omega^2 K^2 - \omega^2 LC + RG = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}$$

$$(\omega^2(2K^2 - LC) + RG)^2 = (R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)$$

$$\omega^4(2K^2 - LC)^2 + R^2G^2 + 2\omega^2(2K^2 - LC)RG = R^2G^2 + \omega^4 L^2 C^2 + \omega^2(L^2G^2 + R$$

Comparing the coefficients of ω^4 & ω^2

$$(2K^2 - LC)^2 = L^2C^2$$

$$4K^4 + \cancel{L^2C^2} - 4K^2 LC = \cancel{L^2C^2}$$

$$2K^2(K^2 - LC) = 0, \quad \therefore K = 0$$

$$\text{or } K = \sqrt{LC}$$

$$L^2G^2 + R^2C^2 = 2RG(2K^2 - LC)$$

$$L^2G^2 + R^2C^2 = 2RG(2LC - LC) = 2RGLC$$

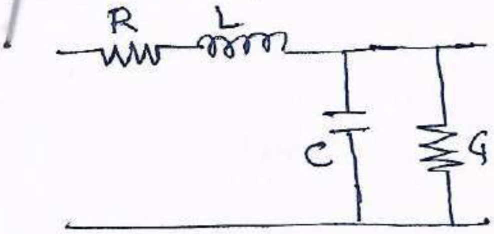
$$\therefore L^2G^2 + 2RGLC + R^2C^2 = 0$$

$$\text{i.e. } (LG - RC)^2 = 0$$

$$LG = RC \qquad \text{i.e. } \cancel{L/G}\ \boxed{R/L = G/C}$$

So for Distortionless T.L. $\Longrightarrow$

Lossy



$$V(x) = V^+ e^{-\gamma x} + V^- e^{+\gamma x}$$

$$I(x) = \frac{V^+ e^{-\gamma x} - V^- e^{+\gamma x}}{Z_0}$$

$$\gamma = \alpha + j\beta$$

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

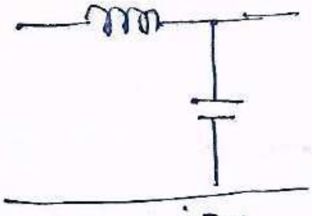
$$Z_0 = \sqrt{\left(\frac{R+j\omega L}{G+j\omega C}\right)}$$

$$Z(x) = \frac{V(x)}{I(x)} = Z_0\left(\frac{e^{-\gamma x} + \Gamma_L e^{+\gamma x}}{e^{-\gamma x} - \Gamma_L e^{+\gamma x}}\right)$$

$$\Gamma(x) = \frac{V^-(x)}{V^+(x)} = \frac{V^- e^{+\gamma x}}{V^+ e^{-\gamma x}} = \frac{V^-}{V^+} e^{2\gamma x}$$

$$\Gamma(x) = \Gamma_L e^{2\gamma x}$$

Lossless



$$V(x) = V^+ e^{-j\beta x} + V^- e^{+j\beta x}$$

$$I(x) = \frac{V^+ e^{-j\beta x} - V^- e^{+j\beta x}}{Z_0}$$

$$\gamma = j\beta$$

$$= j\omega\sqrt{LC}$$

$$\beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{L/C}$$

$$Z(x) = Z_0\left(\frac{e^{-j\beta x} + \Gamma_L e^{+j\beta x}}{e^{-j\beta x} - \Gamma_L e^{+j\beta x}}\right)$$

$$\Gamma(x) = \frac{V^-}{V^+} e^{2j\beta x}$$

$$= \Gamma_L e^{2j\beta x}$$

Standing Waves. – When two waves of exactly same magnitude & frequency travel opposite to each other, the result is not a wave but an "oscillation with no propagation" called as standing wave. Which has fixed location.

$A\cos(\omega t - \beta x)$ $\qquad$ $A\cos(\omega t + \beta x)$

$$Ae^{-j\beta x} + Ae^{+j\beta x} = 2A\cos\beta x$$

Time domain $2A\cos(\beta x)\cdot\cos\omega t$ – ①

... is not wave but a time ...