$$V = V^+ e^{-\gamma x} + V^- e^{+\gamma x} \quad \& \quad I = \frac{V^+}{Z_0} e^{-\gamma x} - \frac{V^-}{Z_0} e^{+\gamma x}$$

$$\cosh \gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2} \qquad \sinh \gamma x = \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$\cosh \gamma x - \sinh \gamma x = \frac{1}{2}\left[\cancel{e^{\gamma x}} + e^{-\gamma x} - \cancel{e^{\gamma x}} + e^{-\gamma x}\right] = e^{-\gamma x}$$

$$\cosh \gamma x + \sinh \gamma x = e^{+\gamma x}$$

$$V = V^+ (\cosh \gamma x - \sinh \gamma x) + V^- (\cosh \gamma x + \sinh \gamma x)$$

$$V = \cosh \gamma x \,(V^+ + V^-) - \sinh \gamma x \,(V^+ - V^-)$$

Similarly . $$I = \frac{1}{Z_0}\left[\cosh \gamma x (V^+ - V^-) - \sinh \gamma x (V^+ + V^-)\right]$$

At input side i.e. at $x = 0$ $V = V_s$ & $I = I_s$

$\therefore$ $\cosh \gamma x = 1$ & $\sinh \gamma x = 0$

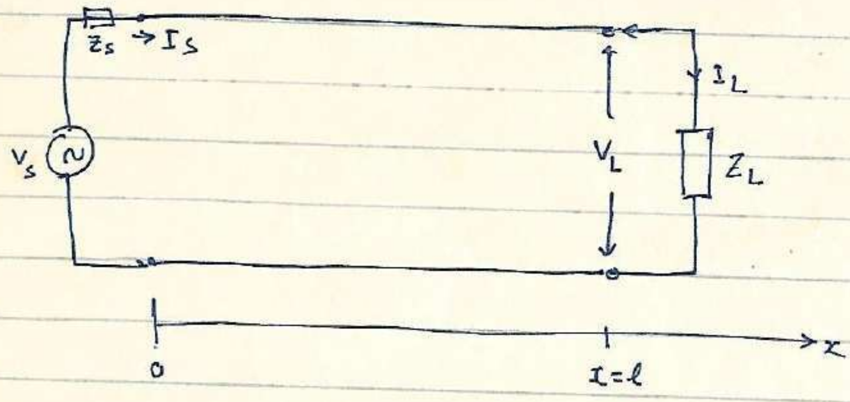
$$V_s = V^+ + V^- \quad \& \quad I_s = \frac{V^+ - V^-}{Z_0}$$

$$\boxed{\begin{aligned} V &= V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x \\ I &= I_s \cosh \gamma x - \left(\frac{V_s}{Z_0}\right) \sinh \gamma x \end{aligned}}$$

$$Z_{in}(x) = Z_0 \cdot \frac{e^{-\gamma l} + \Gamma_L e^{+\gamma l}}{e^{-\gamma l} - \Gamma_L e^{+\gamma l}} \quad \text{where } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in}(x) = Z_0 \cdot \left[\frac{Z_L + Z_0 \tanh \gamma x}{Z_0 + Z_L \tanh \gamma x}\right]$$

OPEN-CIRCUITED T/L



$$V_L = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l$$

$$I_L = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l$$

For open-circuited T/L, $I_L = 0$

$$0 = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l$$

$$Z_{oc} = \frac{V_s}{I_s} = Z_0 \frac{\cosh \gamma l}{\sinh \gamma l}$$

$$\boxed{Z_{oc} = Z_0 \coth \gamma l} \quad - (1)$$

For short-circuited T/L, $V_L = 0$

$$0 = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l$$

$$Z_{sc} = \frac{V_s}{I_s} = \frac{Z_0 \sinh \gamma l}{\cosh \gamma l}$$

$$\boxed{Z_{sc} = Z_0 \tanh \gamma l} \quad - (2)$$

Multiplying (1) & (2) $\quad Z_{oc} \cdot Z_{sc} = Z_0 \coth \gamma l \cdot Z_0 \tanh \gamma l = Z_0^2$

$$\boxed{Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}}}$$

Dividing (2) by (1)

$$\frac{Z_{sc}}{Z_{oc}} = \frac{Z_0 \tanh \gamma l}{Z_0 \coth \gamma l} = \tanh^2 \gamma l$$

$$\boxed{\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

Open & Short Circuited Lossless Transmission Lines:

We have :
$$V_L = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l$$
$$I_L = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l$$

For lossless: $\alpha = 0$
$$\gamma = \alpha + j\beta = j\beta$$

$$V_L = V_s \cosh j\beta l - I_s Z_0 \sinh j\beta l$$

$\because \cosh j\beta l = \cos \beta l$

$\sinh j\beta l = j \sin \beta l$

$$\therefore V_L = V_s \cos\beta l - j \frac{I_s}{\cancel{Z_0}} Z_0 \sin\beta l$$
$$\& \quad I_L = I_s \cos\beta l - j \frac{V_s}{Z_0} \sin\beta l$$

$$\boxed{\begin{aligned} V_L &= V_s \cos\beta l - j I_s Z_0 \sin\beta l \\ I_L &= I_s \cos\beta l - j \frac{V_s}{Z_0} \sin\beta l \end{aligned}}$$

For Open-Circuited T/L : $I_L = 0$

$$0 = I_s \cos\beta l - j\frac{V_s}{Z_0} \sin\beta l$$

$$Z_{oc} = \frac{V_s}{I_s} = \frac{Z_0 \cos\beta l}{j \sin\beta l} \times \frac{j}{j}$$

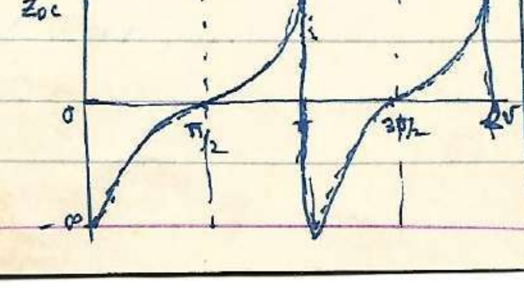
$$\boxed{Z_{oc} = -j Z_0 \cot \beta l}$$

$\beta = \frac{2\pi}{\lambda} \cdot l$

$= \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}$

$= \frac{\pi}{2}$

$\beta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}$



For Short-Circuited T/L : $V_L = 0$

$$0 = V_s \cos\beta l - j I_L Z_0 \sin\beta l$$

$$Z_{sc} = \frac{V_s}{I_s} = j \frac{Z_0 \sin\beta l}{\cos\beta l}$$

$$\boxed{Z_{sc} = j Z_0 \tan\beta l}$$

$l = 0 \quad Z_{sc} = 0$

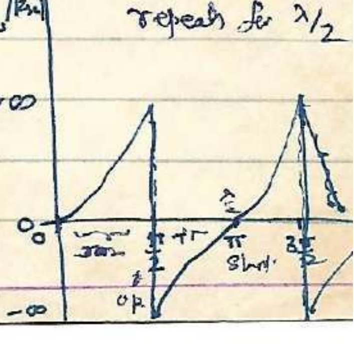
$0 \leq Z_s \leq \frac{\pi}{2}$ $\quad Z_{sc}$ pve img. ↑
$(\lambda/4)$ $\quad$ 'L'

at $l = \lambda/4 \Rightarrow Z_{sc} = \infty$ open

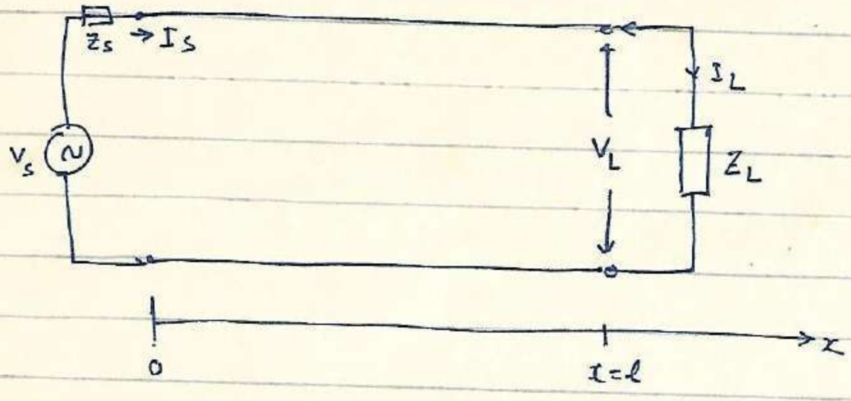
$\frac{\lambda}{4} - \lambda/2 \Rightarrow Z_{sc} = -j$ + capa.

$\lambda/2 - \lambda$

repeats for $\lambda/2$



OPEN-CIRCUITED T/L



$$V_L = V_s \cosh\gamma l - I_s Z_0 \sinh\gamma l$$

$$I_L = I_s \cosh\gamma l - \frac{V_s}{Z_0}\sinh\gamma l$$

For open-circuited T/L, $I_L = 0$

$$0 = I_s \cosh\gamma l - \frac{V_s}{Z_0}\sinh\gamma l$$

$$Z_{oc} = \frac{V_s}{I_s} = Z_0\frac{\cosh\gamma l}{\sinh\gamma l}$$

$$\boxed{Z_{oc} = Z_0 \coth\gamma l} \quad -①$$

For short-circuited T/L, $V_L = 0$

$$0 = V_s \cosh\gamma l - I_s Z_0 \sinh\gamma l$$

$$Z_{sc} = \frac{V_s}{I_s} = \frac{Z_0 \sinh\gamma l}{\cosh\gamma l}$$

$$\boxed{Z_{sc} = Z_0 \tanh\gamma l} \quad -②$$

Multiplying ① & ② $\quad Z_{oc}\cdot Z_{sc} = Z_0\coth\gamma l \cdot Z_0 \tanh\gamma l = Z_0^2$

$$\boxed{Z_0 = \sqrt{Z_{oc}\cdot Z_{sc}}}$$

Dividing ② by ①

$$\frac{Z_{sc}}{Z_{oc}} = \frac{Z_0 \tanh\gamma l}{Z_0 \coth\gamma l} = \tanh^2\gamma l$$

$$\boxed{\tanh\gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

$$V(x) = V^+(x) + V^-(x) \qquad V^+(x) = V_0^+ e^{-\gamma x} \quad V^-(x) = V_0^- e^{+\gamma x}$$

$$= V_0^+ e^{-\gamma x} + V_0^- e^{+\gamma x} \qquad \Gamma_L = \frac{V_0^-}{V_0^+} \qquad \Gamma = \frac{V_0^-}{V_0^+} e^{2\gamma x}$$

$$= V_0^+ \left(e^{-\gamma x} + \Gamma_L e^{\gamma x} \right) \qquad V_0^- = \Gamma_L V_0^+ \qquad = \Gamma_L e^{2\gamma x}$$

$$I(x) = \frac{V_0^+}{Z_0} \left(e^{-\gamma x} - \Gamma_L e^{+\gamma x} \right)$$

$$V(-l) = V_0^+ \left[e^{j\beta l} + \Gamma_L e^{-j\beta l} \right]$$

$$I(-l) = \frac{V_0^+}{Z_0} \left[e^{j\beta l} - \Gamma_L e^{-j\beta l} \right]$$

$$\therefore \quad Z(x) = Z_0 \cdot \frac{e^{-\gamma x} + \Gamma_L e^{\gamma x}}{e^{-\gamma x} - \Gamma_L e^{\gamma x}}$$

$$\boxed{Z(-l) = Z_0 \cdot \frac{e^{\gamma l} + \Gamma_L e^{-\gamma l}}{e^{\gamma l} - \Gamma_L e^{-\gamma l}}}$$

$$\boxed{Z_{in} = Z_0 \left(\frac{Z_L + Z_0 \tanh \gamma x}{Z_0 + Z_L \tanh \gamma x} \right)}$$

$$\gamma = \alpha + j\beta = j\beta$$

$$Z(-l) = Z_0 \cdot \frac{e^{j\beta l} + \Gamma_L e^{-j\beta l}}{e^{j\beta l} - \Gamma_L e^{-j\beta l}}$$

$$\boxed{\begin{aligned} e^{+j\beta l} &= \cos\beta l + j \sin\beta l \\ e^{-j\beta l} &= \cos\beta l - j \sin\beta l. \end{aligned}}$$ Euler's eqn.

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right) \qquad Z_{in} = Z_0 \left(\frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l} \right)$$

Special cases.

$l = \frac{\lambda}{2}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$

$\cos\beta l = \cos\pi = 1$ & $\sin\beta l = 0$

$\therefore Z_{in} = Z_L$ independent of Z_0 or β.

(2) $l = \frac{\lambda}{4}$, $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$\cos\beta l = \cos\pi/2 = 0$ & $\sin\beta l = 1$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

$Z_L = 0$ short

$Z_{in} = \infty$ open

& vice-versa.

(3) $Z_L = jX_L$ Load is purly Reactive.

$$Z_{in} = Z_0 \left(\frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l} \right)$$

$$= Z_0 \left(\frac{jX_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + j^2 X_L \sin\beta l} \right)$$

$$= \ldots \left(X_L \cos\beta l + Z_0 \sin\beta l \ldots \right)$$

If transmission line is electrically small, its length l is small w.r.t. signal wavelength λ

(4) $l << \lambda$

$\beta l = \frac{2\pi}{\lambda} l = 2\pi \cdot \frac{l}{\lambda} \approx 0$

$\cos\beta l = \cos 0 = 1$

& $\sin\beta l = \sin 0 = 0$

$$\left. \begin{aligned} V(-l) &= V(0) \\ I(-l) &= I(0) \end{aligned} \right\} \text{if } l << \lambda$$

$$\boxed{V(-l) = V_0^+ + \Gamma_L V_0^+}$$

$V(0) =$

$Z_{in} = Z_L$

(3) if $Z_L = Z_0$

$Z_{in} = Z_0$

i.e. if load imp. is equal to Z_0 then i/p imp of T.L. will also be Z_0 regardless of length of transmission line.

If load is purly reactive then, input impedance will also be purely ...

Impedance Matching

i) Quarter Wave transformer (Impedance Inverter)

ii) Stub matching.

$$Z_{in} = Z_0 \frac{e^{+\gamma l} + \Gamma_L e^{-\gamma l}}{e^{\gamma l} - \Gamma_L e^{-\gamma l}}. \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$\cosh \gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2}, \quad \sinh \gamma x = \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$\therefore \boxed{Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}.}$$

if $Z_L = 0$ short

$Z_{in} = Z_0 \tanh \gamma l = jZ_0 \tan \beta l$

For lossless xmn line $\gamma = j\beta$

$\tanh j\beta l = \tan \beta l. \quad \sinh j\beta l = j \sin \beta l \quad \cosh j\beta l = \cos \beta l$

$\tan j\beta l = \frac{\sinh j\beta l}{\cosh j\beta l} = \frac{j \sin \beta l}{\cos \beta l}$

$= j \tan \beta l.$

$$Z_{in} = Z_0 \cdot \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

If $l = \frac{\lambda}{4}, \ \beta = \frac{2\pi}{\lambda}$

βl = electrical length $= \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi/2$

$$Z_s = Z_0 \left[\frac{Z_L + jZ_0 \tan \pi/2}{Z_0 + jZ_L \tan \pi/2} \right]$$

$\sin \pi/2 = 1$

$\cos \pi/2 = 0$

$\therefore \tan \pi/2 = \frac{1}{0} = \infty$

$$= Z_0 \left[\frac{Z_L \cos \pi/2 + jZ_0 \sin \pi/2}{Z_0 \cos \pi/2 + jZ_L \sin \pi/2} \right]$$

$$= Z_0 \left[\frac{Z_L \cdot 0 + jZ_0 \cdot 1}{Z_0 \cdot 0 + jZ_L \cdot 1} \right]$$

$$= Z_0 \left[\frac{\cancel{j} Z_0}{\cancel{j} Z_L} \right] = \frac{Z_0^2}{Z_L}$$

$$\boxed{Z_s = \frac{Z_0^2}{Z_L}} \qquad Z_s \propto \frac{1}{Z_L}$$

Input imp is inversly proportional to Load imp

if $Z_s \downarrow$, $Z_L \uparrow$ vice-versa.

if Z_0 is resistive

if Z_L is Capacitive $\rightarrow Z_s$ inductive

& vice-versa.

$\lambda/4$ line acts as step-up or step down impedance transformer