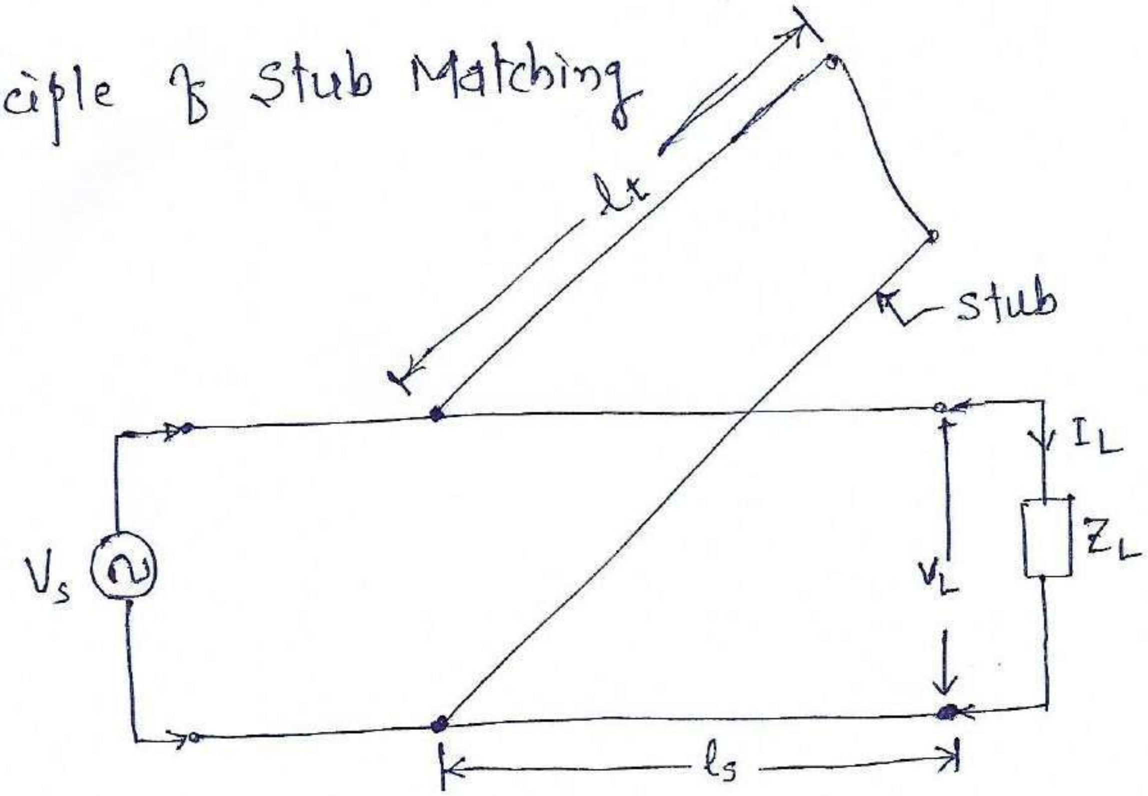
Stub Matching

A Section of open or short Circuited transmission line is called as stub.

Principle of Stub Matching



Stub is connected at a point on main transmission line where input conductance at that point is equal to the characteristics conductance of the line and the stub length is adjusted to provide a Susceptance which is equal in value but opposite in sign to the input Susceptance of main line at that point, so that total Susceptance at that point of attachment is Zero.

Stub matching works on principle of Anti-Resonance.

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Stub Matching

$$Z_{in} = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$Z = R + jX$$

$$Y = G + jB$$

$$Y_{in} = Y_s = \frac{Y_L + jY_0 \tan\beta l}{Y_0 + jY_L \tan\beta l}$$

Normalized w.r.t. Y_0

$$\frac{Y_s}{Y_0} = \frac{Y_L/Y_0 + j\tan\beta l}{1 + j\,Y_L/Y_0 \tan\beta l}$$

let $\frac{Y_s}{Y_0} = Y_s'$ & $\frac{Y_L}{Y_0} = Y_L'$

$$Y_s' = \frac{Y_L' + j\tan\beta l}{1 + jY_L' \tan\beta l} \quad \text{--- (1)}$$

Rationalizing eq. (1)

$$Y_s' = \frac{Y_L' + j\tan\beta l}{1 + jY_L' \tan\beta l} \times \frac{1 - jY_L' \tan\beta l}{1 - jY_L' \tan\beta l} = \frac{Y_L' - jY_L'^2 \tan\beta l + j\tan\beta l - j^2 Y_L' \dots}{1 + (Y_L' \tan\beta l)^2}$$

$$= \frac{Y_L'(1 - j^2 \tan^2\beta l) + j\tan\beta l\,(1 - Y_L'^2)}{1 + (Y_L' \tan\beta l)^2} = G_s' + jB_s'$$

$$= \frac{Y_L'(1 + \tan^2\beta l) + j\tan\beta l\,(1 - Y_L')^2}{1 + (Y_L' \tan\beta l)^2} = G_s' + jB_s' \quad \text{--- (2)}$$

Equating Real & imaginary parts

$$G_s' = \frac{Y_L'(1 + \tan^2\beta l)}{1 + (Y_L' \tan\beta l)^2} \qquad B_s' = \frac{\tan\beta l\,(1 - Y_L'^2)}{1 + (Y_L \tan\beta l)^2}$$

But for No Reflection

$$Y_s' = G_s' + jB_s' = 1 + j0$$

$$Y_s' = G_s' = 1$$

Thus stub has to be connected where there is no reflection

$$G_s' = \frac{Y_L'(1+\tan^2 \beta l)}{1+(Y_L' \tan\beta l)^2} = 1$$

$$1 + Y_L'^2 \tan^2 \beta l_s = Y_L'(1+\tan^2 \beta l_s)$$

$$Y_L'^2 \tan^2 \beta l_s (Y_L'-1) = Y_L'-1$$

$$\tan^2 \beta l_s = \frac{1}{Y_L'} = \frac{1}{Y_L^{o}/Y_o}$$

$$\tan \beta l_s = \frac{1}{\sqrt{Y_L/Y_o}} \quad \text{--- (1)}$$

$$\beta l_s = \tan^{-1}\left(\sqrt{Y_o/Y_L}\right)$$

$$l_s = \frac{1}{\beta}\tan^{-1}\left(\sqrt{Y_o/Y_L}\right)$$

$$\boxed{l_s = \frac{1}{\beta}\tan^{-1}\left(\sqrt{\frac{Z_L}{Z_o}}\right)}$$ This gives location of stub.

$$B_s' = \frac{B_s}{Y_o} = \frac{\tan\beta l_s (1 - Y_L'^2)}{1+(Y_L' \tan\beta l_s)^2}$$

$$\tan\beta l_s = \sqrt{Y_o/Y_L} \quad \text{— from eq (1)}$$

$$\frac{B_s}{Y_o} = \frac{\sqrt{\frac{Y_o}{Y_L}} \cdot \left(1 - \frac{Y_L^2}{Y_o^2}\right)}{1 + \frac{Y_L^2}{Y_o^2} \cdot \frac{Y_o}{Y_L}}$$

$$B_S = \sqrt{\left(\frac{Y_0}{Y_L}\right) \cdot (Y_0 - Y_L)}$$

For loss-less s.c. stub

$$Z_t = j Z_0 \tan \beta l_t$$

or $$Y_t = \frac{1}{j Z_0 \tan \beta l_t} \times \frac{j}{j}$$

$$Y_t = G_t + j B_t = -j Y_0 \cot \beta l_t$$

Susceptance at pt. of attachment is obtained by equating $G_t = 0$

$$B_t = -j Y_0 \cot \beta l_t$$

$$B_S + B_t = 0$$

$$\sqrt{\frac{Y_0}{Y_L}} (Y_0 - Y_L) + \{-j Y_0 \cot \beta l_t\} = 0$$

$$\cot \beta l_t = \frac{Y_0 - Y_L}{Y_0} \sqrt{\frac{Y_0}{Y_L}} = (Y_0 - Y_L) \cdot \frac{1}{\sqrt{Y_0 \cdot Y_L}}$$

$$\tan \beta l_t = \frac{\sqrt{Y_0 Y_L}}{Y_0 - Y_L}$$

$$\boxed{l_t = \frac{\lambda}{2\pi} \cdot \tan^{-1} \frac{\sqrt{Z_L \cdot Z_0}}{Z_L - Z_0}}$$

Stub Matching

Length (l_t) & Position (l_s) of Stub in Terms of Reflection Coefficient (Γ_L)

$$Z_{in}(x) = \frac{V(x)}{I(x)} = Z_0 \frac{e^{-\gamma x} + \Gamma_L e^{\gamma x}}{e^{-\gamma x} - \Gamma_L e^{\gamma x}}$$

$x = -l$

$$Z_{in} = Z_0 \left[\frac{e^{-\gamma x} + \Gamma_L e^{+\gamma x}}{e^{\gamma x} - \Gamma_L e^{\gamma x}}\right] = Z_0$$

We know that -

$$Z_{in} = Z_0 \left[\frac{1 + \Gamma_L e^{-2\gamma l}}{1 - \Gamma_L e^{-2\gamma l}}\right]$$

For high freq T.L. $\alpha = 0$ & hence $\gamma = j\beta$

Also reflection coefficient Γ_L is in general complex quantity & given by

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

where ϕ = Angle of reflection coefficient.

$$\therefore \underset{(Z_s)}{Z_{in}} = Z_0 \left[\frac{1 + |\Gamma_L| e^{j\phi} \cdot e^{-2j\beta l}}{1 - |\Gamma_L| e^{j\phi} \cdot e^{-2j\beta l}}\right]$$

$$\underset{(Z_s)}{Z_{in}} = Z_0 \left[\frac{1 + |\Gamma_L| e^{j(\phi - 2\beta l)}}{1 - |\Gamma_L| e^{j(\phi - 2\beta l)}}\right] \quad - (1)$$

As stub is connected in parallel, so it is better to use admittance

$$Y_s = \frac{1}{Z_s} = \text{Input Admittance} = G_s + jB_s$$

G_s = i/p conductance
B_s = i/p susceptance

$$Y_0 = \frac{1}{Z_0} = G_0 + jB_0.$$

$$\therefore \text{eq. (1) becomes } Y_s = \frac{1}{Z_s} = \frac{1}{Z_0}\left[\frac{1 - |\Gamma_L| e^{j(\phi - 2\beta l)}}{1 + |\Gamma_L| e^{j(\phi - 2\beta l)}}\right]$$

$$\therefore Y_s = G_0 \left[\frac{1 - |\Gamma_L| e^{j(\phi - 2\beta l)}}{1 + |\Gamma_L| e^{j(\phi - 2\beta l)}}\right] \quad - (2)$$

It is assumed that Chara. impe. is resistive $Z_0 = R_0 = \frac{1}{G_0}$

Convert eq. (2) in polar form. i.e. $e^{j\theta} = \cos\theta + j\sin\theta$.

$$Y_s = G_0 \left[\frac{1 - |\Gamma_L| \{\cos(\phi - j\beta l) + j\sin(\phi - 2\beta l)\}}{1 + |\Gamma_L| \{\cos(\phi - 2\beta l) + j\sin(\phi - 2\beta l)\}}\right]$$

Let $\phi' = \phi - 2\beta l$

$$\therefore \; Y_s = G_0 \left[\frac{1 - |\Gamma_L| \cos\phi' - |\Gamma_L| j \sin\phi'}{1 + |\Gamma_L| \cos\phi' + |\Gamma_L| j \sin\phi'} \right]$$

Again Let $|\Gamma_L| \cos\phi' = A$

$|\Gamma_L| \sin\phi' = B$

$$\therefore \; Y_s = G_0 \left[\frac{(1-A) - jB}{(1+A) + jB} \right]$$

Now rationalizing $\; Y_s = G_0 \frac{[(1-A) - jB]}{[(1+A) + jB]} \times \frac{[(1+A) - jB]}{[(1+A) - jB]}$

$$Y_s = G_0 \left[\frac{1 - A^2 - 2jB - B^2}{1 + A^2 + B^2 + 2A} \right]$$

Putting values, we get.

$$Y_s = G_0 \left[\frac{1 - |\Gamma_L|^2 \cos^2\phi' - 2|\Gamma_L| j \sin\phi' - |\Gamma_L|^2 \sin^2\phi'}{1 + |\Gamma_L|^2 \{\cos^2\phi' + \sin^2\phi'\} + 2|\Gamma_L| \cos\phi'} \right]$$

$$Y_s = G_s + jB_s = \frac{G_0 \{[1 - |\Gamma_L|^2 - 2j|\Gamma_L| \cdot \sin\phi']\}}{[1 + |\Gamma_L|^2 + 2|\Gamma_L| \cos\phi']}$$

$$\therefore \; G_s = \frac{G_0 [1 - |\Gamma_L|^2]}{[1 + |\Gamma_L|^2 + 2|\Gamma_L| \cos\phi']} \quad - (A) \qquad \& \qquad B_s = \frac{G_0 \{-2|\Gamma_L| \sin\phi'\}}{[1 + |\Gamma_L|^2 + 2|\Gamma_L| \cos\phi']} \quad - (B)$$

$\therefore$ At point of Attachment of stub, for No reflection to take place (perfect matching) $Z_s = Z_0$ at $l = l_s$

$$\frac{1}{G_s} = \frac{1}{G_0} \quad \text{or} \quad \frac{G_s}{G_0} = 1 \;\text{ at } l = l_s$$

eq. (A) $$1 = \frac{1 - |\Gamma_L|^2}{1 + |\Gamma_L|^2 + 2|\Gamma_L| \cos\phi'}$$

Put $\phi' = \phi - 2\beta l$
$= \phi - 2\beta l_s$

$$1 = \frac{1 - |\Gamma_L|^2}{1 + |\Gamma_L|^2 + 2|\Gamma_L| \cos(\phi - 2\beta l_s)}$$

$$\cancel{1} - |\Gamma_L|^2 = \cancel{1} + |\Gamma_L|^2 + 2|\Gamma_L| \cos(\phi - 2\beta l_s)$$

$$-\cancel{2}|\Gamma_L|^{\cancel{2}} = \cancel{2}|\Gamma_L| \cos(\phi - 2\beta l_s)$$

$$-|\Gamma_L| = \cos(\phi - 2\beta l_s) \quad \text{OR} \quad (\phi - 2\beta l_s) = \cos^{-1}(-|\Gamma_L|)$$