But $\cos^{-1}(-|\Gamma_L|) = \cos^{-1}|\Gamma_L| - \pi$ — (B)

hence $\phi - 2\beta l_s = \cos^{-1}|\Gamma_L| - \pi$ — (3)

$$2\beta l_s = \phi + \pi - \cos^{-1}|\Gamma_L|$$

$$l_s = \frac{\phi + \pi - \cos^{-1}|\Gamma_L|}{2\beta}$$

$$\boxed{l_s = \frac{\lambda}{4\pi}\left[\phi + \pi - \cos^{-1}|\Gamma_L|\right]}$$

Again from eq. (B) the shunt susceptance –

$$\frac{B_s}{G_0} = \frac{-2|\Gamma_L|\sin\phi'}{1+|\Gamma_L|^2+2|\Gamma_L|\cos\phi'} = \frac{-2|\Gamma_L|\sin(\phi-2\beta l_s)}{1+|\Gamma_L|^2+2|\Gamma_L|\cos(\phi-2\beta l_s)} \quad -(4)$$

Put eq. (3) in eq (4)

$$\frac{B_s}{G_0} = \frac{-2|\Gamma_L|\sin[\cos^{-1}|\Gamma_L| - \pi]}{1+|\Gamma_L|^2+2|\Gamma_L|(-|\Gamma_L|)} =$$

$$\boxed{\frac{B_s}{G_0} = \frac{-2|\Gamma_L|\sin[\cos^{-1}|\Gamma_L|]}{1+|\Gamma_L|^2-2|\Gamma_L|^2}} \quad -(5)$$

In order to find $\sin\{\cos^{-1}|\Gamma_L|\}$, Let us put $\cos^{-1}|\Gamma_L| = \theta$

$$\therefore |\Gamma_L| = \cos\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-|\Gamma_L|^2}$$

$\therefore$ eq.(5) $\quad \dfrac{B_s}{G_0} = \dfrac{-2|\Gamma_L|\sin\theta}{1+|\Gamma_L|^2-2|\Gamma_L|^2} = \dfrac{-2|\Gamma_L|\cdot\sqrt{1-|\Gamma_L|^2}}{1-|\Gamma_L|^2}$

$$\boxed{B_s = G_0\left[\frac{-2|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}}\right]} \quad (C)$$

Now if B_t is susceptance of stub & B_s line sus. then in order to cancel the susceptance, both must be zero at point of attachment.

$$\therefore \quad B_s + B_t = 0$$

$$B_t = -B_s = -\left[G_0 \frac{\{-2|\Gamma_L|\}}{\sqrt{1-|\Gamma_L|^2}}\right]$$

$$\text{OR} \quad B_t = G_0\left[\frac{2|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}}\right]$$

But susceptance of short-circuited stub is –

$$B_t = G_0 \cot\beta l_t$$

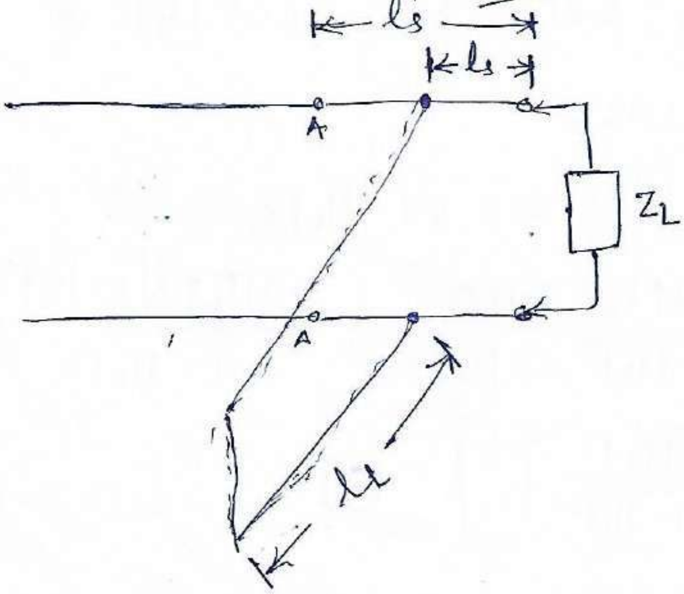
l_t = length of short circuited stub.

$$\cancel{G_0}\cot\beta l_t = \cancel{G_0}\left\{\frac{2|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}}\right\}$$

$$\frac{1}{\tan\beta l_t} = \frac{2|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}} \quad \text{or} \quad \tan\beta l_t = \frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|}$$

$$\beta l_t = \tan^{-1}\frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|}$$

$$\therefore \boxed{l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|}} \quad \text{— (B)}$$



From eq (A'): $\frac{G_s}{G_0} = \frac{1-|\Gamma_L|^2}{1+|\Gamma_L|^2 + 2|\Gamma_L|\cos(\phi - 2\beta l_s)}$

This value of $\frac{G_s}{G_0}$ is maximum when denominator is minimum

i.e. when $\cos(\phi - 2\beta l_s) = -1$

Thus cosine term should be -1 for maximum value of $l_s = l_s'$

$$\cos(\phi - 2\beta l_s') = -1$$

$$\phi - 2\beta l_s' = \cos^{-1}(-1)$$

$$\text{or} \quad \phi - 2\beta l_s' = -\pi$$

$$l_s' = \frac{\phi+\pi}{2\beta} = \frac{\phi+\pi}{2 \times \frac{2\pi}{\lambda}} = \frac{\lambda(\phi+\pi)}{4\pi}$$

$$\boxed{l_s' = \frac{\lambda(\phi+\pi)}{4\pi}}$$

$\therefore$ At a distance l_s' from load the value of

$$\frac{G_s}{G_0} = \frac{1-|\Gamma_L|^2}{1+|\Gamma_L|^2 + 2|\Gamma_L|(-1)} = \frac{(1-|\Gamma_L|)(1+|\Gamma_L|)}{(1-|\Gamma_L|)^2}$$

$$= \frac{\cancel{(1-|\Gamma_L|)}(1+|\Gamma_L|)}{\cancel{(1-|\Gamma_L|)}(1-|\Gamma_L|)} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = VSWR = S$$

$$\frac{G_s}{G_0} = S \qquad \text{But } G_s = \frac{1}{Z_s} = \frac{1}{R_s}, \quad G_0 = \frac{1}{Z_0} = \frac{1}{R_0}$$

$$\frac{1}{Z_s} \cdot Z_0 = S$$

$$\boxed{Z_s = \frac{Z_0}{S}}$$

This is the point of maximum $\frac{G_s}{G_0}$ which is recognized as a point of minimum voltage at a distance of l_s' from load.

At a distance l_s from load, the value of $\frac{G_s}{G_0} = 1$ or $G_s = G_0$. This is point where stub is attached.

Value of distance d is given by

$$d = l_s' - l_s = \frac{\phi+\pi}{2\beta} - \frac{\phi+\pi - \cos^{-1}|\Gamma_L|}{2\beta}$$

$$d = \frac{\cos^{-1}|\Gamma_L|}{2\beta}$$

$$\boxed{d = \frac{\lambda}{4\pi} \cos^{-1}\left(\frac{VSWR-1}{VSWR+1}\right)}$$

This shows that stub should be connected at distance d measure in either direction from voltage minimum point 'A'. Normally st is attached towards load side of that minimum which is nearest to the load voltage standing wave ratio which is existing before the attachment of stub.

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|}$$

But $|\Gamma_L| = \frac{VSWR-1}{VSWR+1} = \frac{S-1}{S+1}$

Putting $l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1-\left|\frac{S-1}{S+1}\right|^2}}{2\left|\frac{S-1}{S+1}\right|}$

$$\boxed{l_t = \frac{\lambda}{4\pi} \tan^{-1} \frac{\sqrt{S}}{S-1}}$$ meters.

If Z_0 is purely Resistive $Z_0 = R_0$

Lossless line: $R=0,\ G=0$

$$\therefore Z_0\Big|_{lossless} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}} \text{ - lossless}$$

For Distortionless : $RC = LG$

$$\text{or } \frac{R}{G} = \frac{L}{C} \quad \text{or} \quad \frac{L}{R} = \frac{C}{G}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R}{G}\left(\frac{1+j\omega L/R}{1+j\omega C/G}\right)} = \sqrt{\frac{R}{G}}$$

$$\boxed{Z_0\Big|_{distortionless} = \sqrt{\frac{L}{C}}}$$

$\therefore$ A lossless line is distortionless but a distortionless line is not always lossless.

Smith-Chart

We know the relation between reflection coefficient & impedance Z of a transmission line as:

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} \quad -① $$

Normalizing eq. ① w.r.t. Z_0

where Γ = reflection coefficient
Z = impedance at any point
Z_0 = Chara. impedance of T.L

$$\Gamma = \frac{Z_N - 1}{Z_N + 1} \qquad \text{where } Z_N = \frac{Z}{Z_0} = r + jx.$$

Thus smith-chart is a plot of reflection coefficient versus normalized impedance of T.L. in Γ-plane as a function of r & x.

$$\Gamma = \frac{Z_N - 1}{Z_N + 1}$$

$$Z_N = r + jx.$$

$$\Gamma = \frac{r + jx - 1}{r + jx + 1} = U + jV$$

Separating into real & imaginary parts.

$$U = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \quad -②$$

$$V = \frac{2x}{(r+1)^2 + x^2} \quad -③$$

Eqns ② & ③, we can obtain two families of Circles, that when superimposed on each other, will make up the entire smith chart.

1. Constant-r circles – Eliminating x from eq. ② & ③ gives

$$\left(U - \frac{r}{r+1}\right)^2 + V^2 = \left(\frac{1}{r+1}\right)^2 \quad -④$$

eq ④ is eqn. of Circle with center located at $(U_0, V_0) = \left(\frac{r}{r+1}, 0\right)$ & radius $R = \left(\frac{1}{r+1}\right)$

1) $r = 0$, $C = (0,0)$, $R = 1$
2) $r = 1$, $C = (0.5, 0)$, $R = 0.5$
3) $r = 2$, $C = (2/3, 0)$, $R = 1/3$
4) $r = \infty$, $C = (1,0)$