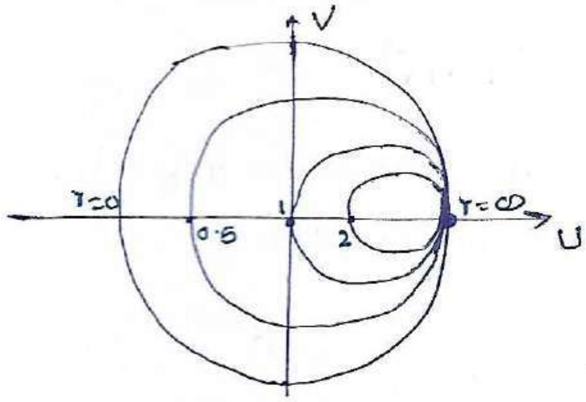


Constant-r-circles.

2. Constant x-circles – Eliminating r from eq. ② & ③ gives

$$(U-1)^2 + \left(V-\frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad - ⑤$$

eq. ⑤ represents constant x-circles with center located at $(U_0', V_0') = \left(1, \frac{1}{x}\right)$

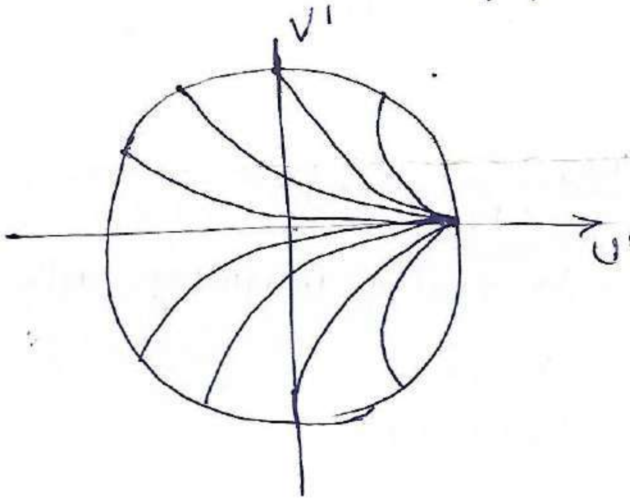
& radius $R' = \frac{1}{|x|}$

1) $x=0$
$C = (1, \infty)$
$r = \infty$

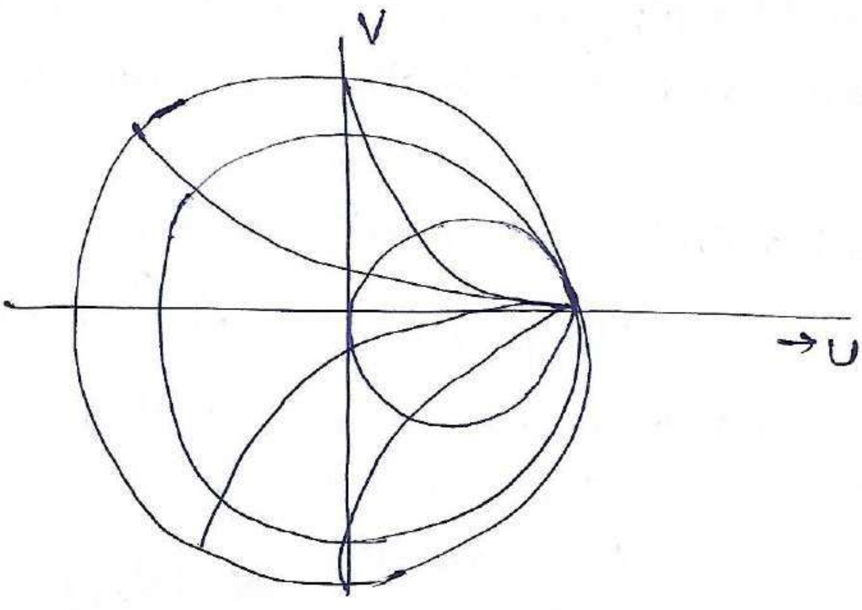
2) $x=1$
$C = (1, 1)$
$r = 1$

3) $x = 2$
$C = (1, 0.5)$
$r = \frac{1}{2} = 0.5$

4) $x = \infty$
$C = (1, 0)$
$r = 0$



When circles r & x obtained in eq. ④ & ⑤ are superimposed it results into Smith-chart shown below.



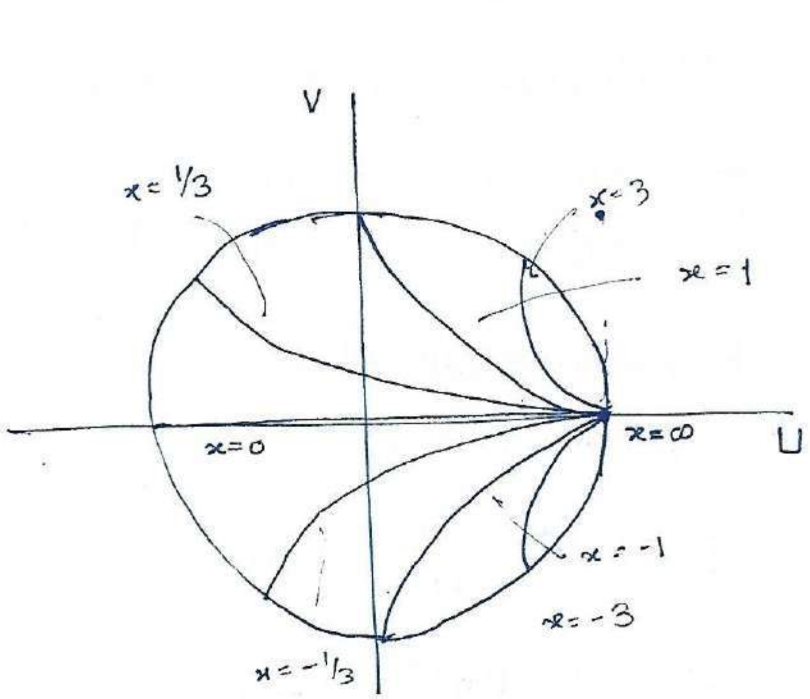
Constant x- Circles

Now eliminating r from eq. ① & ②.

$$(U-1)^2 + \left(V - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

$$(U_0', V_0') = \left(1, \frac{1}{x}\right)$$

$$R = \frac{1}{|x|}$$



1) for $x = \infty$

$U_0', V_0' = (1, 0)$.

$R = \frac{1}{\infty} = 0$

2) for $x = 0$

$U_0', V_0' = (1, \infty)$

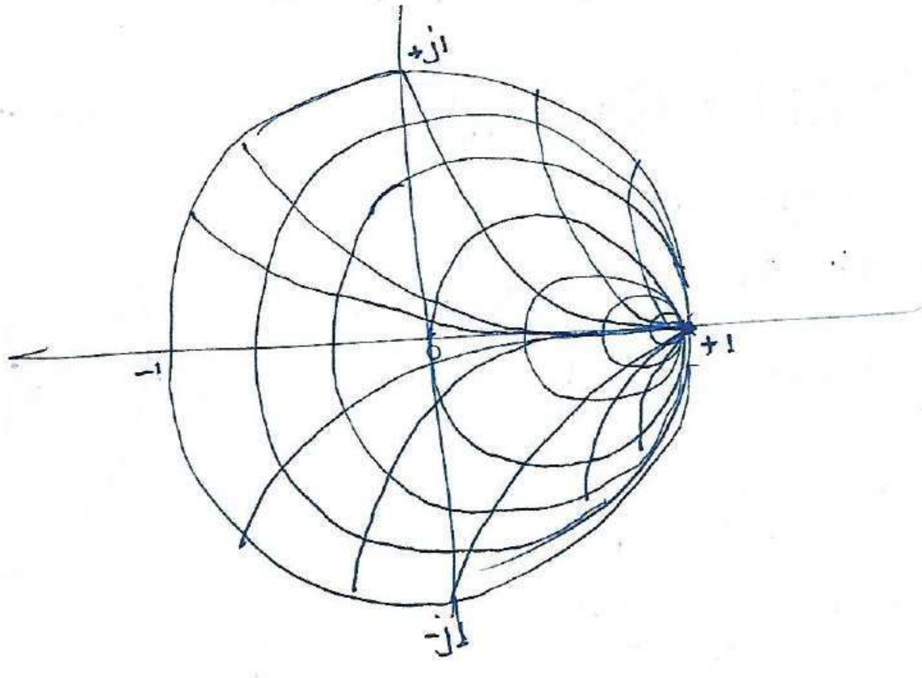
$R = \infty$

3) $x = 1$, $U_0' V_0' = (1, 1)$

$R = 1$

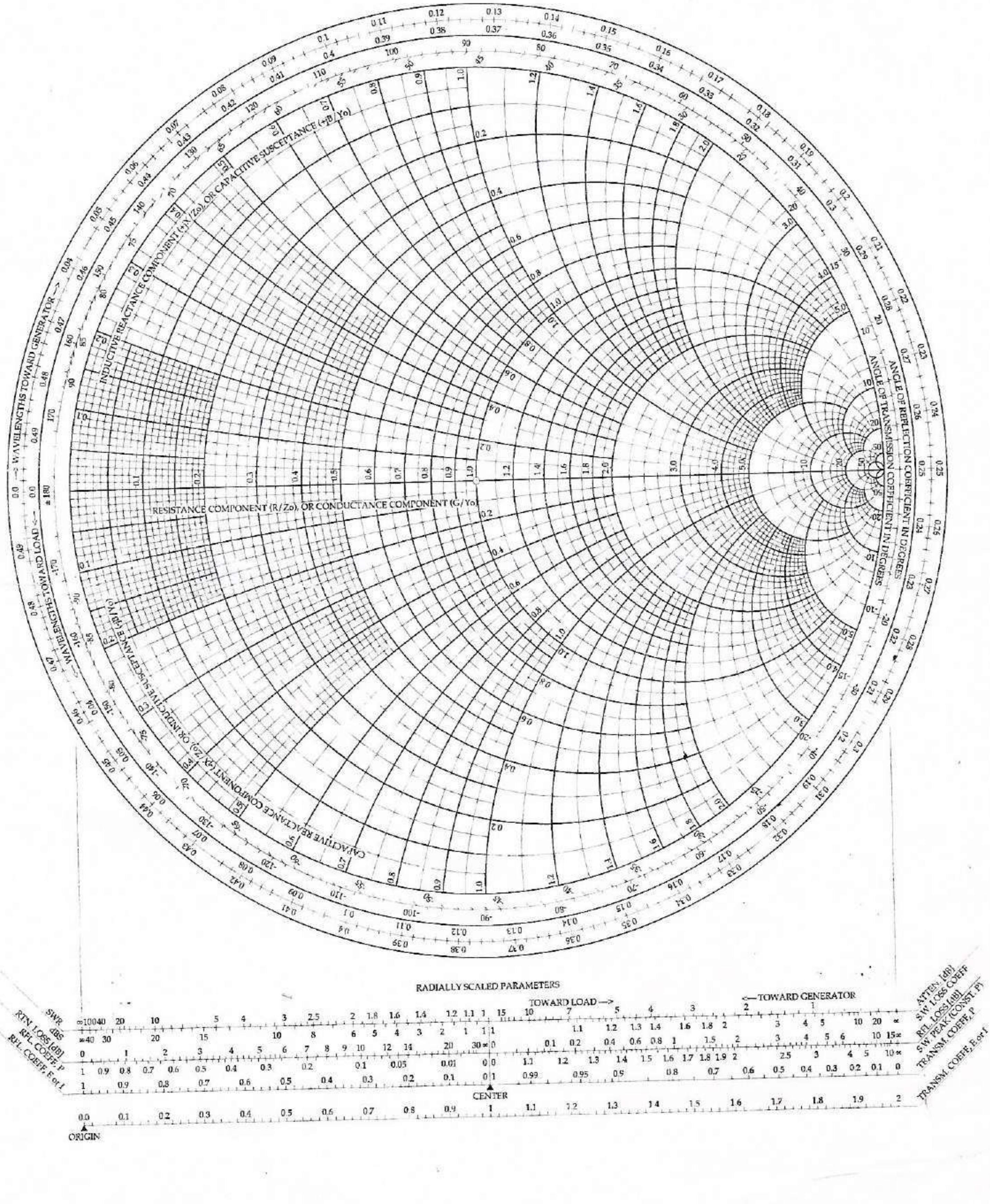
for $x = -1$

$U_0' V_0' = (1, -1$



The Complete Smith Chart

Black Magic Design



Input impedance (Z_{IN}) if Z_L is known.

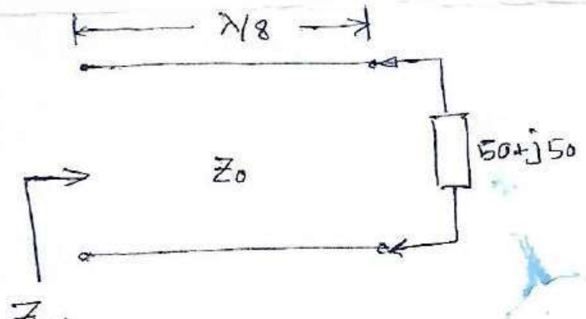
1. Plot normalized load impedance $(Z_L)_N = Z_L/Z_0$ on smith chart
2. Draw the constant VSWR circle that goes thr. $(Z_L)_N$
3. Starting from $(Z_L)_N$, move 'towards generator' on constant VSWR circle a distance l/λ.
4. Read off the normalized i/p impedance value (Z_{IN}/Z_0) from chart.

1. Find the input impedance of a transmission line $Z_0 = 50\Omega$ that has a length of $\lambda/8$ & is connected to a load impedance $Z_L = (50+j50)\Omega$

Sol^n - locate $(Z_L)_N = Z_L/Z_0 = 1+j1$ on the Smith chart.

- Draw constant VSWR circle.
- Move 'towards gen' on constant VSWR circle a dis. $\lambda/8$

$(Z_{IN})_N = 2-j1$ $\quad Z_{IN} = Z_0 . (Z_{IN})_N = (100-j50)\Omega$.



$$Z_{IN} = Z_0 \cdot \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}.$$

$$\beta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$$

- Input impedance using i/p refl. coefficient $|\Gamma_{IN}| \leq 1$

Locate $\Gamma_{IN} = |\Gamma_{IN}| e^{j\theta}$ on Smith chart, the mag. of $|\Gamma|$ can be read off the refl. coeff. vol. radial scale at bottom of the chart while θ is read off the circular scale.

- Normalized values of resistance & reactance (r, x) can be read off the Smith chart at pt. A.

$$Z_{IN} = Z_0 (r + jx)$$

(1) Calculate Z_0, P, V_p at 400kc/s for T.L. having $L = 0.5\,mH/km$, $C = 0.08\,\mu F/km$ negligible R & G

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5\times10^{-3}}{0.08\times10^{-6}}} = 79.08\,\Omega$$

$P = j\beta$ $\quad P = j\omega\sqrt{LC} = j\beta$

& $\beta = \omega\sqrt{LC}$

$$= j2\times3.14\times400\times10^3\sqrt{0.5\times10^{-3}\times0.08\times10^{-6}}$$

$= j15.9$ per km

$|P| = 15.9$ rad/km.

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{0.5\times10^{-3}\times0.08\times10^{-6}}}$$

$= 1.58\times10^5$ km/sec

(2) A lossless T.L. of $100\,\Omega$ chara. Imp. is connected to a load of $200\,\Omega$. Calculate voltage Ref. Coeff. & SWR.

$Z_0 = 100\,\Omega$

$Z_L = 200\,\Omega$

$\Gamma = ?$

$VSWR = ?$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200-100}{200+100} = \frac{1}{3}$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = \frac{4}{2} = 2$$

(3) A. T.L. has following constants $R = 10.4\,\Omega$, $L = 3.66\,mH$, $C = 0.00835\,\mu F$, $G = 0.08\,\mu\mho$. Calculate Z_0, α, β & V_p at $\omega = 5000$ rad/sec.

Sol $Z = R + j\omega L = 10.4 + j18.33 = 21.05\angle 60.4^\circ$

$Y = G + j\omega C = 0.08\times10^{-6} + j41.75\times10^{-6} = 41.77\times10^{-6}\angle 89.9^\circ$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{21.05\angle 60.4^\circ}{41.77\times10^{-6}\angle 89.9^\circ}} = 224.5\angle -14.5^\circ$$

$P = \sqrt{ZY} = 0.007561 + j0.02863$

$$V_p = \frac{\omega}{\beta} = \frac{5000}{0.02863} = 1.746\times10^5\ \text{km/sec}$$

(4) A. T.L. of Chara. imp. 600 Ω is terminated by a reactance of j150 Ω, find input imp. of sec^n 25cm long at freq. of 300 MHz.

Sol^n $Z_0 = 600\,\Omega$, $Z_L = j150\,\Omega$

$l = 25\text{cm} = \frac{25}{100} = \frac{1}{4}\text{m}$, $f = 300\text{MHz}$

$$\lambda = \frac{300}{300} = \lambda = 1\text{m}.$$

$$\beta l = \frac{2\pi}{\lambda}\cdot\frac{1}{4} = \frac{2\pi}{1}\cdot\frac{1}{4} = \pi/2$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$$

$$= Z_0\cdot\frac{Z_0}{Z_L} = \frac{600\times 600}{j150}\times\frac{j}{j}$$

$$Z_{in} = -j2400\,\Omega$$

(5) A 50 Ω lossles T.L. has pure reactance of j100 Ω as its load. Find VSWR in the line.

Sol^n. $Z_0 = 50\,\Omega$ $\quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{j100 - 50}{j100 + 50} = \frac{j2-1}{j2+1}$

$Z_L = j100\,\Omega$

$$|\Gamma_L| = \sqrt{\frac{4+1}{4+1}} = \sqrt{\frac{5}{5}} = 1$$

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

(6) A lossless T.L. having 50 Ω chara imp. & length λ/4 is short circuited at one end & connected to an ideal volt. source of 1V at other end. The current drawn from volt source is — $Z_0 = 50\,\Omega$, $l = \lambda/4$, $V_s = 1V$, $Z_L = 0$ (S.C.)

$$I_s = \frac{V_s}{Z_{in}}$$

For lossless λ/4 line $\quad Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{0} = \infty$

$$I_s = \frac{1}{\infty} = 0\,A$$