

- ① An open wire r.f. transmission line, which may be regarded as loss free has a characteristic impedance $Z_0 = 600 \Omega$ & is connected to a resistive load of 75Ω . Find the position & the length of a short-circuited stub, of the same construction as the line, which would enable the main length of the line to be correctly terminated at a frequency of 150 MHz .

Soln $Z_0 = 600 \Omega$, $Z_L = 75 \Omega$, $f = 150 \text{ MHz}$

$l_s = ?$, $l_t = ?$

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_L}{Z_0}} = \frac{1}{\beta} \tan^{-1} \sqrt{\frac{Z_L}{Z_0}}$$

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{1}{8}}$$

$\lambda/2$, $\lambda = \frac{c}{f} = \frac{300}{150} = 2 \text{ m}$.

$$= \frac{2}{2\pi} \tan^{-1} \sqrt{1/8} = \frac{2}{2\pi} \times 19.44^\circ \times \frac{\pi}{180} = 0.108 \text{ m}$$

$l_s = 10.8 \text{ cm}$.

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_L Z_0}}{Z_L - Z_0} = \frac{1}{\pi} \tan^{-1} \left(-\frac{8.484}{21} \right) = \frac{1}{\pi} \tan^{-1}(-0.404)$$

$$= \frac{1}{\pi} \cdot (-22^\circ) = \frac{1}{\pi} \cdot +158 \times \frac{\pi}{180} = \frac{158}{180} = 0.8790 \text{ m} = \underline{\underline{87.9 \text{ cm}}}$$

- ② Calculate the position & length of a short-circuited stub designed to match a 200Ω load to a transmission line whose characteristic impedance is 300Ω

$Z_0 = 300 \Omega$, $Z_L = 200 \Omega$

$l_s = ?$, $l_t = ?$

$$l_s = \frac{\lambda}{2\pi} \tan^{-1}(0.8164) = \frac{\lambda}{2\pi} \cdot 39.22^\circ \times \frac{\pi}{180} = \frac{39.22 \lambda}{360} = 0.10894 \lambda \text{ m}$$

$$= 10.894 \lambda \text{ cm}$$

$$l_t = \frac{\lambda}{2\pi} \tan^{-1}(-2.4495) = \frac{\lambda}{2\pi} \cdot (-67.8^\circ)$$

$$= \frac{\lambda}{2\pi} (180 - 67.8^\circ) \times \frac{\pi}{180} = \frac{\lambda}{2\pi} \cdot 112.2 \times \frac{\pi}{180}$$

$= 0.3116 \lambda \text{ m}$

$= \underline{\underline{31.16 \lambda \text{ cm}}}$

DATA #220000
Date: 11/1/2024

- ① A two wire line has a char. imp. of 300Ω . It is used at 100MHz . Find the minimum length required of a short circuited line that can be used to simulate an inductive reactance of 200Ω .

$$Z_L = 200\Omega, Z_0 = 300\Omega, f = 100\text{MHz}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{100 \times 10^6} = 3\text{m}$$

$$\lambda = \frac{300}{100} = 3\text{m}, \checkmark$$

The min. length of short ckt. line

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_L - Z_0}{Z_L + Z_0}} = \frac{3}{2\pi} \tan^{-1} \sqrt{\frac{200 - 300}{200 + 300}}$$

$$= \frac{3}{2\pi} \tan^{-1} \sqrt{\frac{-100}{500}} = \frac{3}{2\pi} \tan^{-1}(-0.4472)$$

$$= \frac{3}{2\pi} (-47.8^\circ) = \frac{3}{2\pi} (-0.837\text{ rad}) \approx -\frac{\pi}{180}$$

$$= \frac{12.2}{180} = 0.068\text{m}$$

- ② At what distance from the aerial end of a line of char. imp 500Ω should a stub be introduced on it so as to match the line with the aerial of 80Ω input imp., radiating waves both 100m .

$$Z_0 = 500\Omega, Z_L = 80\Omega, \lambda = 100\text{m}, l_s = ?$$

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_L - Z_0}{Z_L + Z_0}} = \frac{100}{2\pi} \tan^{-1} \sqrt{\frac{80 - 500}{80 + 500}}$$

$$= \frac{50}{\pi} \tan^{-1} \sqrt{-0.16} = \frac{50}{\pi} \tan^{-1}(-0.4)$$

$$= \frac{50}{\pi} (-21.8^\circ) \approx -\frac{\pi}{180} = \frac{5 \times 21.8}{18} = \frac{109}{18} = 6.055\text{m}$$

1) An UHF lossless T.L. working at 1 GHz is connected to an unmatched load producing a voltage reflection coefficient of $0.5 \angle 30^\circ$. Calculate, after deriving necessary relations, length & position of a single stub to match the line.

$$\rightarrow f = 1 \text{ GHz}, \quad \Gamma = 0.5 \quad \phi = 30^\circ$$

$$l_s = ?, \quad l_t = ?$$

$$\lambda = \frac{300}{1000} = \frac{c}{f} = 0.3 \text{ m.}$$

$$\phi = 30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6} \text{ radians.}$$

$$l_s = \frac{\lambda}{4\pi} [\phi + \pi - \cos^{-1} |\Gamma|] = \frac{\lambda}{4\pi} \left[\frac{\pi}{6} + \pi - \cos^{-1}(0.5) \right]$$

$$l_s = \frac{\lambda}{4\pi} \left[\frac{\pi}{6} + \pi - 60^\circ \right] = \frac{\lambda}{4\pi} \left[\frac{\pi}{6} + \pi - 60 \cdot \frac{\pi}{180} \right] = \frac{\lambda}{4\pi} \left[\frac{\pi}{6} + \pi - \frac{\pi}{3} \right]$$

$$\left[\frac{7\pi}{6} - \frac{\pi}{3} \right]$$

$$l_s = \frac{\lambda}{4\pi} \left[\frac{7\pi}{6} - \frac{\pi}{3} \right] = \frac{\lambda}{4\pi} \left[\frac{5\pi}{6} \right] = \frac{0.3}{4} \times \frac{5}{6} = \frac{1.5}{24}$$

$$\boxed{l_s = 0.0625 \text{ meters.}}$$

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1 - |\Gamma|^2}}{2|\Gamma|} = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1 - (0.5)^2}}{2 \times 0.5}$$

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{0.75} = \frac{\lambda}{2\pi} \tan^{-1}(0.8660) = \frac{\lambda}{2\pi} \cdot 40.9^\circ$$

$$l_t = \frac{\lambda}{2\pi} \times 40.9 \times \frac{\pi}{180}$$

$$l_t = \frac{0.3}{2\pi} \times \frac{40.9 \times \pi}{180} = \frac{12.27}{360}$$

$$\boxed{l_t = 0.03408 \text{ meters.}}$$

$$\frac{10.51}{1040}, \frac{10.38}{1031}$$

A 300Ω T.L. feeding an antenna has a standing wave ratio of 4. & the distance from the load & the first voltage minima is 28cm. If the frequency is 150MHz, design a single stub matching system to eliminate standing wave ratio from the maximum possible length of the line.

$$\begin{aligned} \rightarrow Z_0 &= 300\Omega & f &= 150\text{MHz} \\ VSWR &= 4 & l_s &= ? \\ \gamma_{\min} &= 28\text{cm} & l_t &= ? \end{aligned}$$

The first voltage minima is

$$\lambda = \frac{c}{f} = \frac{300}{150}$$

$$\lambda = 2\text{meters}$$

$$2\beta\gamma_{\min} - \phi = \pi$$

$$2 \times \frac{2\pi}{\lambda} \cdot \frac{28}{100} - \phi = \pi$$

$$2 \times \frac{2\pi}{\lambda} \cdot \frac{28}{100} - \pi = \phi$$

$$\frac{4\pi}{2} \cdot \frac{28}{100} - \pi = \phi$$

$$\pi \left(\frac{28}{50} - 1 \right) = \phi$$

$$\phi = \pi \left(-\frac{11}{25} \right) = -3.14 \times 0.44 \text{ or } -0.44\pi \text{ radians.}$$

$$\Gamma_L = \frac{VSWR - 1}{VSWR + 1} = \frac{4 - 1}{4 + 1} = \frac{3}{5} = 0.6$$

$$\begin{aligned} \cos^{-1}(0.6) &= 53.1^\circ \\ \frac{53.1 \times \pi}{180} &= 0.925\pi \end{aligned}$$

$$\begin{aligned} l_s &= \frac{\lambda}{4\pi} (\phi + \pi - \cos^{-1}|\Gamma_L|) = \frac{2}{4\pi} (-0.44\pi + \pi - \cos^{-1}(0.6)) = \frac{2}{4\pi} (0.56\pi - 0.925\pi) \\ &= \frac{2}{4\pi} = \frac{1}{2\pi} (0.265\pi) = 0.1325 \end{aligned}$$

$$\boxed{l_s = 0.1325\text{meters}}$$

$$\begin{aligned} l_t &= \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1 - |\Gamma_L|^2}}{2|\Gamma_L|} = \frac{2}{2\pi} \tan^{-1} \frac{\sqrt{1 - (0.6)^2}}{2 \times 0.6} = \frac{1}{\pi} \tan^{-1} \frac{\sqrt{0.64}}{1.2} = \frac{1}{\pi} \tan^{-1} \frac{0.8}{1.2} \\ &= \frac{1}{\pi} \tan^{-1}(0.666) = \frac{1}{\pi} \cdot 33.69^\circ = \frac{1}{\pi} \cdot 33.69 \times \frac{\pi}{180} = \frac{33.69}{180} \\ &= 0.1872\text{meters} \end{aligned}$$

- ① Find the input Impedance of a transmission line $Z_0 = 50 \Omega$ that has a length of $\lambda/8$ & is connected to a load impedance $Z_L = (50 + j50) \Omega$

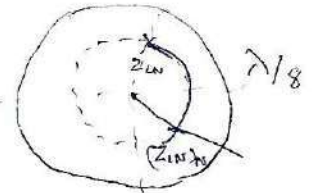
Step 1: locate $(Z_L)_N = Z_L/Z_0 = 1 + j1$ on Smith chart

2. Draw a constant VSWR circle as shown in fig.

3. Now, move towards gen. on const. VSWR circle a distance of $\lambda/8$ to obtain

$$(Z_{IN})_N = 2 - j1$$

$$Z_{IN} = 50(2 - j1) = (100 - j50) \Omega$$



- ② Find the admittance value for an impedance value of $Z = (50 + j50) \Omega$ in a 50Ω system.

$$Z_0 = 50 \Omega$$

Step 1: Locate $Z_{IN,N} = 1 + j1$ on Smith Chart.

2. Draw constant VSWR circle.

3. Read 180° away on the const. VSWR circle.

$$Y_N = 0.5 - j0.5$$

$$Y_0 = 1/Z_0 = 1/50 = 0.02$$

$$Y = Y_0 Y_N = 0.02(0.5 - j0.5)$$

$$Y = 0.01 - j0.01$$

- ③ A transmission line with char. imp. of $Z_0 = 50 \Omega$ is terminated into the following load impedances

$$Z_L = 0 \text{ (Short Ckt.)}, Z_L = \infty \text{ (Open Ckt.)}, Z_L = 50 \Omega$$

$$Z_L = (16.67 - j16.67) \Omega, Z_L = (50 + j150) \Omega$$

Find the individual ref. Coefficient & display them on Smith Chart

1) $\Gamma = -1$

2) $\Gamma = +1$

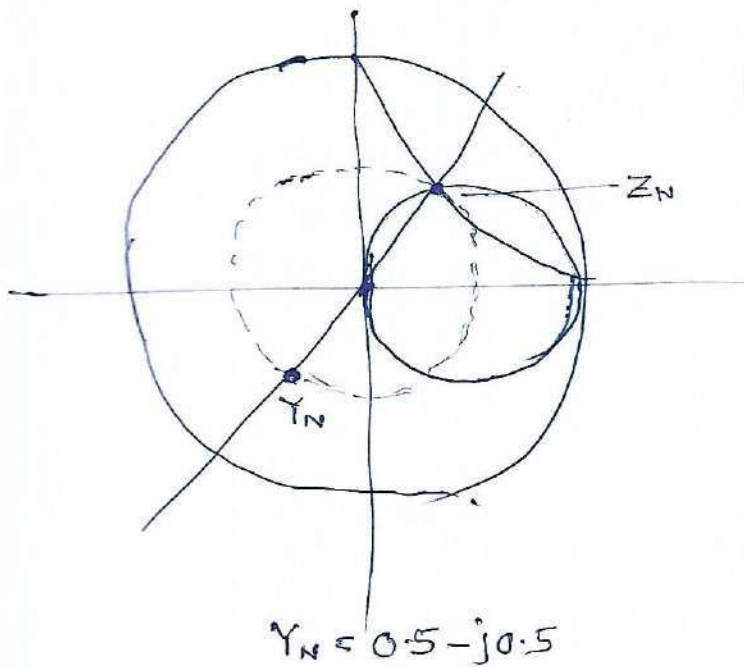
3) $\Gamma = 0$

4) $\Gamma = 0.54 \angle 22.1^\circ$

5) $\Gamma = 0.82 \angle 34^\circ$

① Determination of Admittance from Impedance.

$$Z = (50 + j50) \Omega, Z_0 = 50 \Omega$$



$$Y_0 = \frac{1}{Z_0} = \frac{1}{50} = 0.02 S$$

$$Y = Y_0 Y_N = (0.5 - j0.5) \times 0.02$$

② A T.L. with chara imp. of $Z_0 = 50 \Omega$ is terminated into the following load impedances.

- i) $Z_L = 0$ (short ckt.)
- ii) $Z_L = \infty$ (open ckt.)
- 3) $Z_L = 50 \Omega$
- 4) $Z_L = 16.67 + j16.67$
- 5) $Z_L = 50 + j150 \Omega$

Find individual ref coefficients Γ & display them on Smith chart.

$$\frac{Z_L - Z_0}{Z_L + Z_0}$$

- i) $\Gamma = -1$
- 2) $\Gamma = 1$
- 3) $\Gamma = 0$
- 4) $\Gamma = 0.54 \angle 22.1^\circ$
- 5) $\Gamma = 0.83 \angle 34^\circ$

