JUNE-JULY, 2023 EXAMINATION **Code: 1006**

M.SC. EXAMINATION

MA 94208 Real and Complex Analysis

Time :3 hour **Max. Marks: 70**

TOTAL NO. OF QUESTIONS IN THIS PAPER : 5

Note: Each question carry five subparts a , b , c , d and e. Attempt subparts a,b,c and any one from d or e in each question. All questions carry equal marks.

	Questions	Marks	CO	BL	PI
Q.1(a)	If the outer measure of a set is zero then the set is measurable.	02	1	1,2	1.1.1
(b)	The intersection of two measurable set is again a measurable.	02	1	1,2	1.1.1
(c)	Let $f_i = i = 1,2 \ldots \ldots .. n$ be measurable function defined on a measurable set E. Then $\max\{f_1, f_2 \ldots\ldots\ldots f_n\}$ & $\min\{f_1, f_2 \ldots\ldots\ldots f_n\}$ are measurable.	03	1	1,2	1.1.1
(d)	Define Lebesgue integral & prove that let f be bounded function defind on [a,b]. If f is Riemann integrable on [a,b] then it is Lebesgue integrable on [a,b] & $R\int_a^b f(x)dx = \int_a^b f(x)dx$	07	1	1,3	1.1.1
	OR				
(e)	State & Prove Lebesgue bounded convergence theorem.	07	1	1.2	1.1.1
Q.2 (a)	Find b_n of the function $f(x) = x + x^2 \; for \; -\pi < x < \pi$.	02	2	1,2	1.1.1
(b)	Write the Dirichlet's Condition for a Fourier series .	02	2	1,2	1.1.1
(c)	Define convergence of fourier series for continuous & piecewise continuous function.	03	2	1,2	1.1.1
(d)	Find the Fourier Series for the function $f(x) = x \quad -\pi < x < 0$; $-x \quad 0 < x < \pi$ And hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$	07	2	3	1.1.1
	OR				
(e)	Find the Fourier Series for the function $f(x) = x\,cosx \quad -\pi < x < \pi$	07	2	1,3	1.1.1
Q.3 (a)	Test the analyticity of the function $w = sinz$ and hence derive that : $\frac{d}{dz}(sinz) = cosz$	02	3	1,2	1.1.1
(b)	Find the harmonic conjugate function of the function $U(x,y) = 2x(1-y)$.	02	3	2,3	1.1.1
(c)	Determine $\frac{1}{z}$ is analytic or not ?	03	3	1,3	1.1.1
(d)	Show that the function $f(z) = u + iv$, where $f(z) = f(x) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ satisfies C-	07	3	3	1.1.1

	R equations at $z = 0$. Is the function analytic at $z = 0$?				
	OR				
(e)	Evaluate $\int f(z)dz$ where $f(z) = y - x - i3x^2$ from $z = 0$ to $z = 1+i$ along the path (i) from (0 ,0) to A (1 ,0) to B (1,1) (ii) $y = x$	07	3	3	1.1.1
Q.4(a)	What is the difference between Cauchy's integral theorem & Cauchy's integral formula.	02	4	1,2	1.1.1
(b)	Define Conformal mapping & Cauchy's residue theorem. $x^3(1+i) - y^3(1-i)$	02	4	1,2	1.1.1
(c)	Evaluate $\int e^{\sin z^2} dz$ where c is $\|z\| = 1$	03	4	1,2	1.1.1
(d)	State & Prove Cauchy's integral theorem.	07	4	1,2	1.1.1
	OR				
(e)	Evaluate $\int \frac{e^z}{z^2(z+1)^3} dz$ $C: \|z\| = 2$	07	4	2,3	1.1.1
Q.5 (a)	Define Singular points & isolated singularity.	02	5	1,2	1.1.1
(b)	$\int \frac{1}{z-a} dz$ where c is a simple closedcurve and point $z = 0$ (i) outside (ii) inside .	02	5	1,2	1.1.1
(c)	Determine the residue at the points for the function $\frac{z+2}{(z+1)^2(z-2)}$	03	5	2	1.1.1
(d)	Using Cauchy's residue theorem to evaluate. $\int \frac{dz}{(z^2+4)^2}$ where c is the circle $\|z - i\| = 2$	07	5	3	2.4.1
	OR				
(e)	Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ using contour integration.	07	5	3	2.4.1