

which can be written as

$$RC \frac{dp_o}{dt} + p_o = p_i$$

and p_o are considered the input and output, respectively, then the transfer function of the system is

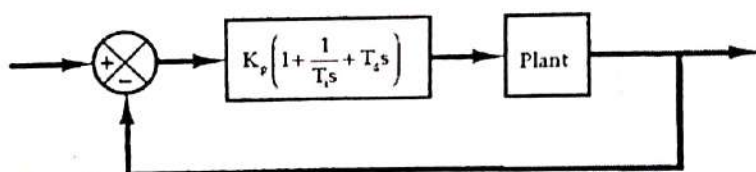
$$\frac{P_o(s)}{P_i(s)} = \frac{1}{RCs + 1}$$

RC has the dimension of time and is the time constant of the system.

ZIEGLER - NICHOLS RULES FOR TUNING PID CONTROLLERS

Control of Plants. Figure shows a PID control of a plant. If a mathematical model of the plant can be obtained, then it is possible to apply various design techniques for determining parameters of the controller that meet the transient and steady-state specifications of the closed-loop system. However, if the plant is so complicated that its mathematical model cannot be easily obtained, then an analytical or computational approach to the design of a PID controller is not possible. Then we must resort to experimental approaches to tuning of PID controllers.

The process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers (meaning to set values K_p , T_i , T_d) based on experimental step responses or based on the value of K_p that results in marginal stability when only proportional control action is used. Ziegler-Nichols rules, which are briefly presented in the following, are useful when mathematical models of plants are not known. (These rules can, of course, be applied to the design of systems with known mathematical models.)



Such rules suggest a set of values of K_p , T_i , and T_d that will give a stable operation of the system. However, the resulting system may exhibit a large maximum over-shoot in the step response, which is not acceptable. In such a case we need series of fine tunings until an acceptable result is obtained. In fact, the Ziegler-Nichols tuning rules give an educated guess for the parameter values and provide a starting point for further tuning, rather than giving the final settings for K_p , T_i , and T_d in a single shot.

Ziegler-Nichols Rules for Tuning PID Controllers

Ziegler and Nichols proposed rules for determining values of the proportional gain K_p , integral time T_i , and derivative time T_d based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on

the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols. They are available in the literature and from the manufacturers of such controllers.)

There are two methods called Ziegler–Nichols tuning rules: the first method and the second method. give a brief presentation of these two methods.

First Method

In the first method, we obtain experimentally the response of the plant to a unit-step input, as shown. If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step curve may look S-shaped, as shown in Figure. This method applies if the response to a step input exhibits an S-shaped curve. Such step-response curves may be generated experimentally or from a dynamic simulation of a plant.

The S-shaped curve may be characterized by two constants, delay time L and time constant T . The delay time and time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line $c(t) = K$, as shown in Figure.

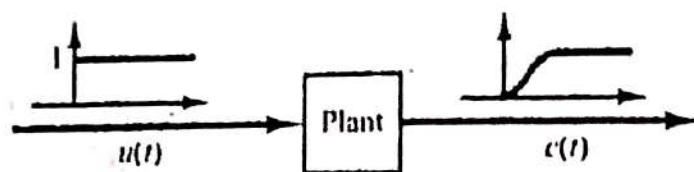


Fig. Unit-step response of a plant

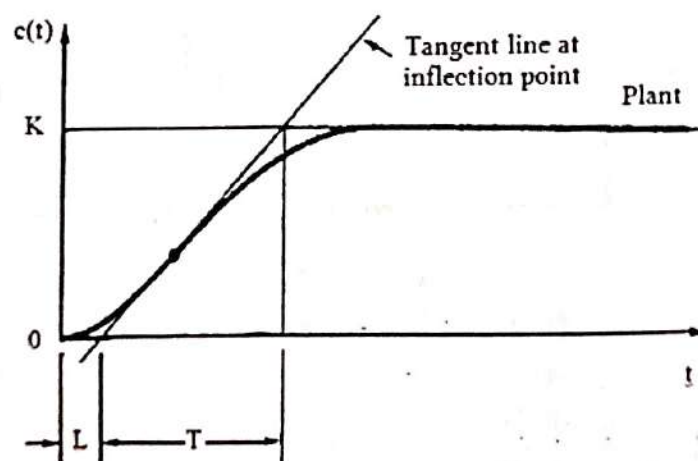


Fig. S-shaped response curve

Table: Ziegler-Nichols Mining Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9 \frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2 \frac{T}{L}$	2Δ	$0.5L$

tion $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{K e^{-Ls}}{Ts + 1}$$

gler and Nichols suggested to set the values of K_p , T_i , and T_d according to the formula shown in Table.

ice that the PID controller tuned by the first method of Ziegler-Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \\ &= 1.2 \frac{T}{L} \left(1 + \frac{1}{2Ls} + 0.5Ls \right) \\ &= 0.6T \frac{\left(s + \frac{1}{L} \right)^2}{s} \end{aligned}$$

us, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

Second Method

the second method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure), increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K may take, then this method does apply.) Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally

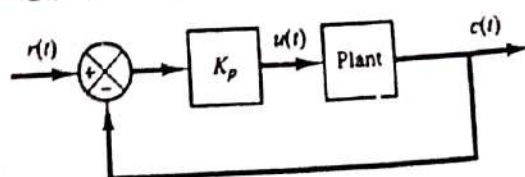


Fig. Closed-loop system with a proportional controller

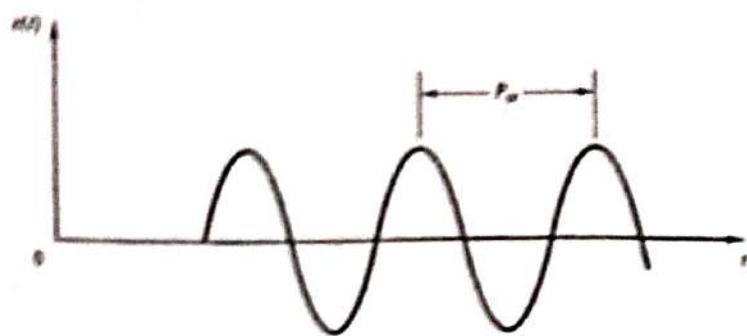


Figure : Sustained oscillation with period P_{cr} (P_{cr} is measured in sec.)

determined (see Figure). Ziegler and Nichols suggested that we set the values of the parameters K_p according to the formula shown in Table.

Table: Ziegler-Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5 K_{cr}$	∞	0
PI	$0.5 K_{cr}$	$\frac{1}{1.2} P_{cr}$	0
PID	$0.6 K_{cr}$	$0.5 P_{cr}$	$0.125 P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler-Nichols rules gives

$$\begin{aligned}
 G_c(s) &= K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \\
 &= 0.6 K_{cr} \left(1 + \frac{1}{0.5 P_{cr} s} + 0.125 P_{cr} s \right) \\
 &= 0.075 K_{cr} P_{cr} \frac{\left(s + \frac{4}{P_{cr}} \right)^2}{s}
 \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

Note that if the system has a known mathematical model (such as the transfer function), then we can use the root-locus method to find the critical gain K_{cr} and the frequency of the sustained oscillations ω_{cr} where $P_{cr} = 2\pi/\omega_{cr}$. These values can be found from the crossing points of the root-locus branches with the $j\omega$ axis. (Obviously, if the root-locus branches do not cross the $j\omega$ axis, this method does not apply.)

Ziegler-Nichols tuning rules (and other tuning rules presented in the literature) have been widely used to tune controllers in process control systems where the plant dynamics are not precisely known. Over many years, these tuning rules proved to be very useful. Ziegler-Nichols tuning rules can, of course, be applied to plants whose dynamics are known. (If the plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available, in addition to Ziegler-Nichols tuning rules).

Example 1.

Consider the control system shown in Figure in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler-Nichols tuning rule for the determination of the values of parameters K_p , T_i and T_d . Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

Since the plant has an integrator, we use the second method of Ziegler-Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

s^3	1	5
s^2	6	K_p
s^1	$\frac{30 - K_p}{6}$	
s^0	K_p	

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CONTROL VALVE & ELECTRIC ACTUATORS

Valve is defined as any device by which the flow of fluid may be started, stopped, or regulated by a movable part that opens or obstructs passage.

Functions

- Start and stop flow
- Regulation of flow
- Back flow prevention
- Release pressure

Valve Capacity

The capacity or flowing rate of a control valve is given by

$$C_v = Q \sqrt{\frac{G}{\Delta P}}$$

where ΔP = pressure drop (psi) across the valve

G = liquid's specific gravity (1 for water)

Q = Flow rate in GPM

The value of C_v is in US GPM

$$\text{In SI unit } C_v = 11.7Q \sqrt{\frac{G}{\Delta P}}$$

where, Q = water flow (m³/hr)

ΔP = pressure drop (kPa)

Rangeability :

is the ratio of maximum controllable flow minimum controllable flow, i.e.,

$$\frac{Q_{\max}}{Q_{\min}} = \frac{(C_v)_{\max}}{(C_v)_{\min}}$$